

*Elizabeth City State University*  
Nurturing ECSU Research Talent(NERT)  
1997-98 Annual Report



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Dr. Linda Hayden, Principal Investigator  
Box 672 ECSU Elizabeth City, NC 27909  
(252) 335-3696 voice (252) 335-3790 fax  
lhayden@ga.unc.edu

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# Table of Contents

Program Description	1
1997-98 Research Teams	2
Summer Internship Report	3
1997-98 Enrollment & GPA Report	4
Graduate Success Report	5
Honors Convocation	6-9
Final Team Reports Flier	10
Final Team Reports	11
Fractals/Chaos Team	
Physics Team	
Multimedia Authoring Team	
Javascript/HTML	
ATM/System Administration Team	
Computer Visualization Team	
Appendix	
Highlights and Photo Pages	





# N.E.R.T.

## Nurturing ECSU Research Talent Elizabeth City State University

This program, entitled "Nurturing ECSU Research Talent" focuses on undergraduate education and undergraduate research experiences. Nurturing these young researchers is a primary concern. Highest priority is given to providing them with the guidance and skills to insure their entrance and success in graduate school. Further, each student learns the fundamentals of scientific research, in a team setting, under the guidance of a faculty mentor. Program activities are as follows:

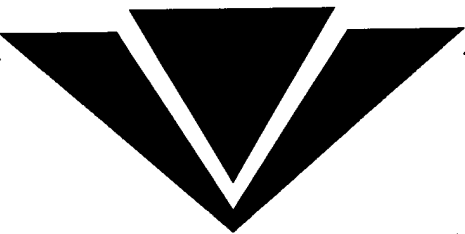
### 1. Student development activities

- a) Recruitment of high ability minority students;
- b) Providing a precollege/summer experience for recruited students;
- c) Providing research experiences;
- d) Providing a mentor, graduate school counseling and GRE preparation;
- e) Providing financial support for students in the form of research scholarships;
- f) Providing funds for student travel.

### 2. Infrastructure activities

- a) Enhancement of current computer graphics and operating systems courses;
- b) Development of a new course in computer visualization;
- c) Establishing a visiting lecture series in computer science;
- d) Providing UNIX network management support;
- e) Acquisition of computer equipment appropriate to support of research training.





## 1997-98 Undergraduate Research Teams

<u>Team Name</u>	<u>Mentors</u>	<u>Team Members (26)</u>
Fractals/Chaos	Dr. D. Sengupta	Corey Ellis, Sr/Math Tammara Ward, Sr/Math Ayonda Moore, Jr/Math
~~~~~		
JAVA	Dr. K Kulkarnie	Angela Mizelle Fr/CS Lakisha Mundon, Jr/Math Sheri Joyner, So/CS Micheal Boisson, Fr/CS Joseph Gale, Fr/CS
~~~~~		
ATM Networks	Dr. L. Hayden Mr. D. Archer Ms. S. Saunders	Curtis Felton, Sr/CS Fred Sessoms, Sr/CS Charles Gatling, Sr/CS Melvin Anderson, Sr/CS Jamaal Turner, Sr/In Tech Laverne Williams, Sr/CS Derrek Burrus, Jr/CS Antonio Rook, Jr/CS Courtney Fields, Jr/CS
~~~~~		
Visualization	Dr. K. Edoh	Kuchumbi Hayden, Jr/CS Alicia Jones So/CS
~~~~~		
Physics	Dr. A. L. Choudhury	Santiel Creekmore, Jr/Phy Katrina Godwin, So/CS Arthur Fenner Sr/Math
~~~~~		
Multimedia	Dr. L. Hayden Mrs. Amie Aydlett	Jonathan Williams, Fr/CS Donald Charity So/Math Je'aime Powell, Fr/CS



# ONR/NASA Summer 1998 Summer Placement/Internship Report

<u>name</u>	<u>class</u>	<u>summer placement/internship</u>
Boisson, Michael	FR	Newport News Naval Shipyard
Gale, Michael	FR	ONR-AASERT Summer Research Program
Mizelle, Angela	FR	ONR-AASERT Summer Research Program
Powell, Je'aime	FR	ONR-AASERT Summer Research Program
Williams, Jonatha	FR	ONR-AASERT Summer Research Program
Charity, Donald	SO	ONR- Naval Research Lab
Godwin, Katrina	SO	NASA-Kennedy Space Craft Center
Jones, Alicia	SO	ONR- Naval Research Lab (Ronald McNair)
Joyner, Sheri	SO	ORISE Dept. of Energy (Ronald McNair)
Burrus, Derrek	JR	U S Coast Guard
Creekmore, Santie	JR	NSU-Institute in Materials Science(Ronald McNair)
Fields, Courtney	JR	Department of Energy
Hayden, Kuchumbi	JR	Ronald McNair Research Program
Moore, Ayonda	JR	Virginia Tech Summer Program(Ronald McNair)
Mundon, Lakisha	JR	Ronald McNair Research Program
Roock, Antonio	JR	University of Alabama Summer Research
Anderson, Melvin	SR	NC A&T Grad School +MSU High Performance Computing
Ellis, Corey	SR	IBM and New Mexico State University Fellowship
Felton, Curtis	SR	NC A&T Grad School + FermiLab
Fenner, Arthur	SR	Dept. of Energy
Gatling, Charles	SR	NC A&T Grad School +MSU High Performance Computing
Sessoms, Fred	SR	IBM
Turner, Jamaal	SR	Dept. of Transportation
Ward, Tammara	SR	IBM
Williams, Laverne	SR	FermiLab + Michigan State Univ. GEM Fellowship

	1997-98 Enrollment and GPA Report										
	Total Enrollment				ONR Enrollment				Total	ONR	
								Graduated	Prof.	Prof.	
								School	School	School	
Major Discipline	FR	SO	JR	SR	FR	SO	JR	SR			
Engineering	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
Biology	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	
Chemistry	3	3	6	4	0	0	0	4	2	0	
Computer Science	63	30	39	29	5	2	3	29	4	4	
Mathematics	5	7	10	15	0	1	2	14	0	0	
Physics	1	1	1	3	0	0	1	1	0	0	
Totals	72	41	56	51	5	3	6	48	6	4	
Class Year	Mean GPA for all				Mean GPA for						
	Students				ONR Students						
Freshman				2.147					3.047		
Sophomore				1.953					3.736		
Junior				3.056					3.204		
Senior				3.183					3.408		

## Graduate Success Program Results

<u>Name</u>	<u>University</u>	<u>Degree Sought/Earned</u>
Jovita Harrell	Hampton	Masters Computer Science
Renee Basnight	Hampton	Masters Computer Science
Chonda Gayle	Hampton	Masters Computer Science
Eva Dail Koltuniak	Hampton	Masters Computer Science
Tim McCray	Hampton	Masters Computer Science
Sharon Saunders	Hampton	Masters Computer Science
Michelle Brown-Emmanual	Hampton	Masters Computer Science
Karen Felton	Duke	Masters Chemistry
Stephanie Vaughan	Hampton	Masters Computer Science
Cathy Thomas	Ohio State	Masters Computer Science
Felicia Bowser	North Carolina State	Masters Computer Science
Clarence Jones	Hampton	Masters Physics
Michael Fields	Hampton	Masters Physics
Bonnie Gardner	Maryland	Masters Computer Science
Stacia McFadden	Michigan State	Masters Computer Science
Cultilda Monk	Fayetteville	Math Education
Kim Gordon	Virginia State	Masters Mathematics
Darnley Archer	Old Dominion	Masters Computer Science
Brian Jordan	Hampton	Masters Mathematics
Belinda Banks	Norfolk State	Masters Communications
Abdula Fofana	Howard	Masters Computer Science
Charles Gatling	North Carolina A&T	Masters Computer Science
Melvin Anderson	North Carolina A&T	Masters Computer Science
Curtis Felton	North Carolina A&T	Masters Computer Science
Laverne Williams	Michigan State	Masters Computer Science
Teresa Bright	Ohio State	Masters Computer Science



## Chancellor's List: 3.75 to 4.0 Average

*Vice-Chancellor's List: 3.50 to 3.74 Average*

*Honor List: 3.00 to 3.49 Average*

Elizabeth City State University

ELIZABETH CITY, NORTH CAROLINA

**MICKEY L. BURNIM, CHANCELLOR**

# Honors Convocation



*Thursday, April 16, 1998  
2:00 o'clock in the afternoon  
Moore Hall Auditorium*

Elizabeth City State University is a constituent institution of  
THE UNIVERSITY OF NORTH CAROLINA

PRELUDE.....Prelude and Fugue on "O Trinity/Kent"  
Dr. Rachel W. Gragson, Organist.....Johannes Brahms

**Program**  
Dr. Lois W. Green, Interim Associate Vice Chancellor for Academic Affairs  
- Presiding -

INVOCATION.....The Reverend Derrick Wilkins  
Manager, ECSU Academic Computing Center  
Graduate, Honors Program

MUSICAL SELECTION....."I Want Jesus To Walk With Me"  
Toneika Stephens, Soprano  
Mr. Billy Hines, Conductor

INTRODUCTION OF SPEAKER.....Mr. Harold V. Lawson  
Junior, Honors Program

ADDRESS.....Dr. Albert L. Walker  
Vice Chancellor for Academic Affairs  
Elizabeth City State University, Elizabeth City, North Carolina

PRESENTATION OF AWARDS.....Dr. Rachel W. Gragson  
Chairman, Honors Council

CONGRATULATIONS.....Miss Samantha L. Brown  
Director, Honors Program

ANNOUNCEMENTS.....Dr. Mickey L. Burnim  
Senior, Honors Program

POSTLUDE.....Miss Bobbie J. Hayman  
Sophomore, Honors Program

**Special Honors Awards**

Certificates Presented by the Honors Program - Awarded to all Students for  
Spring Semester 1996-97 and Fall Semester 1997-98 (as listed)

**CHANCELLOR'S DISTINGUISHED EMBLEM AWARDS**

**Scholar's Blazers**

Felicia Bowser ✓  
Judith Fields  
Rachel Holmes  
Christopher Johnson  
Stacia McFadden ✓  
Phillip Puryear  
Bonnie Stroud

**THE HONORS PROGRAM**

**Certificates of Merit**

Chenay Beamon  
Alayna Benson  
Felicia Best  
Samantha Brown  
Derrick Burris  
Tanisha Darden  
Peter Eley  
Corey Ellis ✓  
Keywonna Everette  
Steven Gilchrist  
Katrina Godwin ✓  
Bobbie Hayman  
Tamara Hedgebeth  
Nicole Hoffer  
Sheri Jovner ✓  
Harold Lawson  
Christie Long  
Lavar Mizelle  
Ayonda Moore ✓  
Lakisha Mordon ✓  
Terrica Nelson  
Jennifer Noorrey  
Shawana Person  
Natasha Peters  
Alisha Reid  
Chantel Reynolds  
Paula Rose  
Felicia Saunders  
Fredrika Simons  
Angel Swinme  
Kenya Thomas  
Blair Todd  
Jarrod Turner  
Lisa Wang  
Laverne Williams ✓

**HONORS PROGRAM DARIN L. COLE AWARD**

Chenay Beamon,  
Derrick Burris, Tanisha Darden,  
Steven Gilchrist, Nicole Hoffer, Harold Lawson

**GREEK HONORS CUP**

Delta Chi Chapter  
Delta Sigma Theta Sorority, Inc.

**ART DEPARTMENT**

Artistic Achievement Award.....Cynthia Dashiell

**BIOLOGY DEPARTMENT**

Clarence E. Biggs Award.....Alisha Reid  
Herman Cooke Research Excellence Award.....Travis Albritton, Tarsha Darden  
Curtis D. Turnage Award.....Tamara Hedgebeth, Alisha Reid  
Freshman Achievement Award in Biology.....Scott Forbes  
Sophomore Achievement Award in Biology.....Joshua Henson  
Betina Holloman

**BUSINESS AND ECONOMICS DEPARTMENT**

Graduating Senior Award.....Jennifer Pugh  
Junior Student-of-the-Year Award.....Rachel Haines  
Excellence in Accounting Award.....Rachel Holmes  
Excellence in Business Education Award.....Shonda Pittman  
Excellence in Economics & Finance Award.....Jenet Jernigan  
Excellence in Management Award.....On Thanh Van  
Excellence in Marketing Award.....Tamika Hinton  
Excellence in Accounting Award.....Angela Jennings  
The Wall Street Achievement Award.....Kenya Thomas  
Wachovia Fund for Excellence Award.....Anetha Coaxum, Stephanie Baily  
Professional Excellence NABA Chapter Award.....Tonya Peterson  
Professional Excellence Award - Phi Beta Lambda.....Eric Powell  
Professional Excellence Award - SIFE.....Stephanie Baily, Jennifer Pugh

**DIVISION OF EDUCATION**

Charles A. Bryant Scholarship.....Travis Albritton  
Lois W. Green Graduating Senior Award in Teacher Education.....Delicia Wright  
Outstanding Senior in Psychology Award.....Dana Golden, Natalie Hall,  
Lakitra Evans, Tyrell Moore, Natasha Peters

Outstanding Academic Performance Award in Elementary Education.....Louise Crosswait, Jonathan Downing, Terrie Byrum,  
Mary Owen, Lisa Pipkin, Angel Swinme  
Outstanding Special Education Student Award.....Brandi Richardson

**EDUCATIONAL TALENT SEARCH PROGRAM**

Academic Excellence Award.....Tyrell Carr, Cynthia Simpson,  
Tisa Stiles, Tracy Taylor  
Exemplary Service Award.....Mardo Moore, Jerome Wilson  
McNair Scholars Eagle Award.....Melvin Anderson, Sabina Butts,  
Santiel Creekmore, Courtney Fields, Charles Gatling,  
Talesh Lane, Ayonda Moore, Charmaine Morgan,  
Terrica Nelson, Jamaal Jorner, Timeka Whitehead

McNair Scholars Challenger Award.....Katrina Godwin, Laverne Williams  
Albert Walker

**GENERAL STUDIES DIVISION**

Division of General Studies Award.....Justin Winslow

**GEOSCIENCES DEPARTMENT**

Freshman/Sophomore Academic Award.....Jennifer Anstutz, Dennis Linner, Sunday Timell  
Junior Academic Award.....Larry Elmore  
Senior Academic Award.....

**INCENTIVE SCHOLARSHIP PROGRAM**

Outstanding Freshman Incentive Scholar.....Joshua Henson  
Outstanding Sophomore Incentive Scholar.....Karen Stokley  
Outstanding Junior Incentive Scholar.....Tarsha Darden  
Outstanding Senior Incentive Scholar.....Rachel Holmes, Angel Swinme

**LANGUAGE, LITERATURE & COMMUNICATION DEPARTMENT**

Graduating Senior Award.....Angela Burris  
E. M. Spellman Award.....Angela Burris  
Viking Yearbook Award.....Kimberly Hines, Ryan Taylor  
Deanna Moring, Albert Walker

**MATHEMATICS & COMPUTER SCIENCE DEPARTMENT**

The S. S. Sachdev Senior Award in Mathematics.....Samantha Brown, Corey Ellis  
The Margaret G. Sharpe Award.....Samantha Brown

The J. L. Houston Senior Award in Computer Science.....	Laverne Williams ✓
The Umfirt E. Locus Sophomore Award in Computer Science.....	Katrina Godwin ✓
NASA-NRTS Achievement Award.....	Marlicia Cranby, Barbara King Vera Powell, Kimberly Wright Melvin Anderson,
ONR: Nurturing ECSU Research Program Award.....	Michael Boisson, Derek Burrus Sentiel Creekmore, Courtney Fields, Kuchumbi Hayden, Angela Mizelle, Lakisha Mundon, Je'aine Powell, Antonia Rook, Jamaal Turner, Tammara Ward, Johnathan Williams ✓
Office of Naval Research Scholars Award.....	Donald Charity, Corey Ellis,* Curtis Felton, Charles Galling, Katrina Godwin, Sheri Joyner, Ayonda Moore, Laverne Williams
Office of Naval Research Award of Excellence.....	Charles Galling, Laverne Williams
MILITARY SCIENCE DEPARTMENT	
Top Scholastic Average Award.....	Cadet Joseph Kurtzweil
Cadet Honors Award.....	Cadet Bobby Burrus
ROTC Academic Excellence Ribbon.....	Cadet Bobby Burrus, Cadet Demetrius Melton, Cadet Robin Williams
MUSIC DEPARTMENT	
Music Department Award.....	Delicia Wright
Charles Penrose Award.....	Benjamin Taylor, Toni Wood
Edna Davis Theory Award.....	Brian Snow
Florence Folkes Lassiter Award.....	Toneika Stephens
PHYSICAL EDUCATION AND HEALTH DEPARTMENT	
Physical Education Academic Achievement Award.....	Heather Biggs, Colin Woodley
PHYSICAL SCIENCES DEPARTMENT	
1998 Outstanding Student Chemist Award.....	Mark Mwaura
Rochelle Cleaners Excellence in Chemistry Award.....	Mark Mwaura
Rochelle Cleaners Excellence in Physics Award.....	Santiel Creekmore ✓
Physical Sciences Academic Excellence Award.....	Santiel Creekmore, Craig Foster, Mark Mwaura, Veronica Overton, Nadiah Shaw, Timeka Whitehead
SOCIAL SCIENCES DEPARTMENT	
Department of Social Sciences Award.....	Tandeka Whitaker
Timothy H. Wamack Scholarship.....	Marlo Moore
History Club Award.....	Marlo Moore, Kelvin Walston
History Excellence Award.....	Debra Eason
Political Science Academic Excellence Award.....	James Martin
Criminal Justice Excellence Award.....	Sheila Gordon, Josephina Spruill
Sociology Excellence Award.....	Jennifer Nooney
Social Work Excellence Award.....	Teresa Garris, Irving Long, Traci Pritchard
Social Work Highest GPA Award (transfer student).....	Traci Pritchard
Social Work Highest GPA Award (non-transfer student).....	Tandeka Whitaker
Social Work Achievement Award.....	Elouise Francis, Ernestine Futrell, Teresa Garris, Irving Long, Traci Pritchard, Tandeka Whitaker, Pamela Williams*
STUDENT AFFAIRS DIVISION	
Davis Cup.....	New Complex
Honda Campus All-Star Challenge Team.....	Accepting -Harold Lawson, Laverne Williams
Henrietta B. Ridley Award for Excellence in Leadership.....	Shaunell McMillan, Deanna Moring, Darnell Woods Aua Opoku
STUDENT SUPPORT SERVICES AWARD.....	Tisa Stiles
TECHNOLOGY DEPARTMENT	
Freshman Achievement Award in Technology.....	Angela Mitchell
Sophomore Achievement Award in Technology.....	Joseph Tillet

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## Junior Achievement Award in Technology Industrial Technology Faculty Award

Roger Iby  
Jamaal Turner ✓

## CLUBS AND ORGANIZATIONAL AWARDS

### The Alpha Kappa Alpha Sorority Scholarship

Delta Theta Chapter.....Blair Todd

### The Alpha Kappa Alpha Scholarship

Charna Cooper

### The Delta Sigma Theta Sorority Scholarship

Elizabeth City Alumnae Chapter.....Ayonda Moore

### Kappa Delta Pi Counselors' Award - Member

Norma Jeffcoat

### Kappa Delta Pi Counselors' Award - Non-Member

Blair Todd

## WHO'S WHO

Travis Albritton  
Tynoshia Barnes  
Chenay Beamon  
Jennifer Beatley  
Alayna Benson  
Heather Biggs  
Samantha Brown  
Angela Burrus  
Derek Burrus  
Annette Cherry  
George Copeland  
Corey Ellis ✓

Shirley Emanuel  
Scott Forbes  
Ernestine Futrell  
Charles Galling ✓  
Rachael Haines  
Nikki Heyward  
Kimberly Hines  
Nicole Hoffer  
Rachel Holmes  
Nathaniel Isaac  
Hope Jennings  
Hope Jones

Charles Lamb  
Telesh Lane  
Chianti Lloyd  
Linda Logan  
Tonya Lyons  
Julie Motta  
Mary Owen  
Natascha Peters  
Cindy Powell  
Alisha Reid  
Diane Ross  
Letitia Roulhac

Fred Sessoms ✓  
Summer Smith  
Toneika Stephens  
Jamaal Turner ✓  
Jarrod Turner  
Nekia Walker  
Rebecca Walston  
August Whidbee  
Tandeka Whitaker  
Laverne Williams  
Delicia Wright

*Honors Spring Semester 1996-97*

## Chancellor's List: 3.75 to 4.0 Average

[illegible]

*Honor List: 3.00 to 3.49 Average*

[illegible]

## *Nurturing ECSU Research Talent (NERT) Program*

Sponsored by  
Elizabeth City State University  
Office of Naval Research

Thursday April 2, 1998 5:00 pm 116 LH

### FRACTALS & CHAOS

Dr. D. Sengupta, Mentor  
Corey Ellis, Sr/Applied Math  
Ayonda Moore, Jr./Applied Math  
Tammara Ward, Sr/Math

### JAVA SCRIPT

Dr. K. Kulkarnie, Mentor  
Anglea Mizelle Fr/CS  
Lakisha Mundon, Jr/Math  
Sheri Joyner, So/CS  
Micheal Boisson, Fr/CS  
Joseph Gale, Fr/CS

### PHYSICS

Dr. L. Choudhury, Mentor  
Santiel Creekmore, Jr/Phy  
Katrina Godwin, So/CS  
Arthur Fenner Sr/Math

Tuesday April 7, 1998 5:00 pm 116 LH

### NETWORKS

Dr. L. Hayden, Mentor  
Ms. S. Saunders, Mentor  
Curtis Felton, Sr/CS  
Derrek Burrus, Jr/CS  
Antonio Rook, Jr/CS  
Fred Sessoms, Sr/CS  
Courtney Fields, Jr/CS  
Charles Gatling, Sr/CS  
Melvin Anderson, Sr/CS  
Jamaal Turner, Sr/Ind Tech  
Laverne Williams, Sr/CS

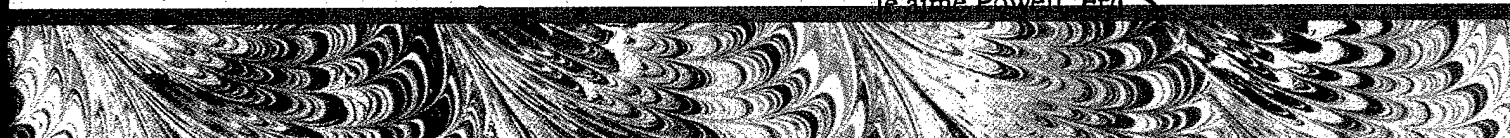
### VISUALIZATION

Dr. K. Edoh, Mentor  
Kuchumbi Hayden, Jr/CS  
Alicia Jones, So/CS

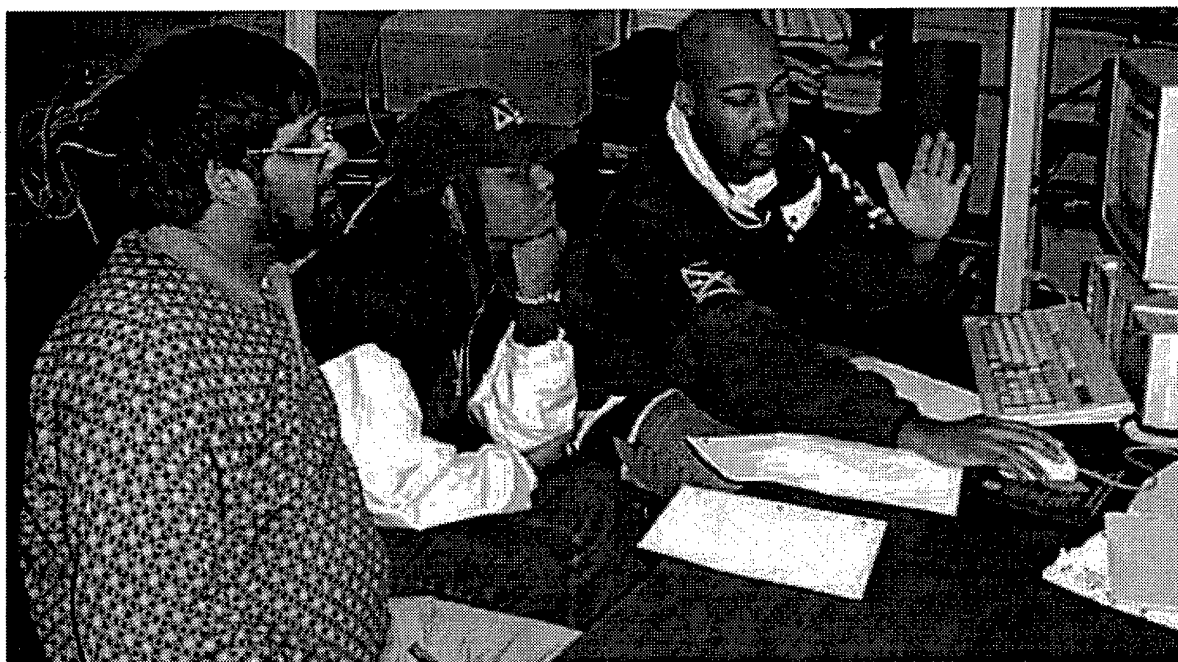
### MULTIMEDIA

Mrs. Amie Aydlett, Mentor  
Jonathan Williams, Fr/CS  
Donald Charity, So/Math  
Le'aima Powell, Fr/CS

ONR Final Research Team Reports



# *Fractals / Chaos & Dynamical Systems*



*Team Mentor: Dr. D. Sengupta*

*Team Members: Ayonda Moore  
Corey Ellis  
Tammara Ward (not shown)*

### Abstract

In this Project we will begin the systemic study of "differential equation". This study will not be a study of an equation and tricks to 'solve' it, but rather of the dynamic 'movement' produced by the differential equation.

A differential equation describes 'how things change', and if we know where we start, we should be able to predict where we go and how fast. The 'systems' we will study here are families of initial value problems (I.V.P.) given by a differential equation and a starting position.

One good analogy for a mathematical dynamical system is the 'flow' on the surface of a smoothly moving river. The differential equations corresponds to the velocity vector at each point on the surface, while the collection of paths of all the water particles constitute the solution flow. In this project we will study numerical, graphical and symbolic description of mathematical flows determined by a differential equation and its initial conditions. This will shed new light by analyzing them geometrically. Geometry is a powerful qualitative tool to answer the 'where we go' questions. Numerics and symbolics answer quantitatively 'how fast questions'.

In this project we will explicitly analyze and solve several models which leads to linear and nonlinear differential equations using Mathematica. We will mainly concentrate on the following issues:

1. What are the assumptions that go into describing change mathematically
2. What dynamic movement does the change law produce?

"A dynamical system is a mathematical way of describing a system that changes [3, pg 1]." In the previous research the approach one used to analyze the dynamical systems was iteration. Now the approach one will be focusing attention on will be differential equations.

### Differential Equations vs. Iteration

Both differential equations and iteration are commonly used to model real life phenomenon i.e., population; and either approach might be applied to the same situation [3, pg 4]. One might think that iteration would be the easier approach to use, particularly since the calculus of the derivatives is avoided. Conceptually this is a correct assumption, but iteration methods do not always predict the motions (orbits) correctly [3, pg 4]. Therefore, another approach must be used for analysis on certain dynamical systems.

### Differential Equations

Differential equations has long been a field in which a mathematician could spend a lifetime. The best understood differential equations are, as one might expect, linear differential equations. Nonlinear equations are far more of a challenge because of the lack of knowledge one has about them.

When solving differential equations analytical, numerical, and graphical techniques can be used. Most of these differential equations in general do not lend themselves to special analytical methods [3, pg 5]. When modeling real-life situations using differential equations, analytical methods for finding solutions is not a recommended approach. The reason is an overall understanding of the model can not be obtained usually with analytical methods. An approach that gives a overall solution to the system is the qualitative (graphical) approach.

Differential equations can now be visualized using computer graphics, but they must be in one of three forms, one-dimensional, two-dimensional, or three-dimensional. The example that was analyzed was a two-dimensional linear dynamical system.

## Qualitative Technique: Slope Fields

Whenever possible, it is useful to have a visual representation for a mathematical problem. This is especially true for differential equations. Slope fields is a method developed for visualizing graphically the graphs of the solutions to differential equations [3, pg 34]:

$$\frac{dy}{dt} = f(t, y)$$

"If the function  $y(t)$  is a solution of the equation  $\frac{dy}{dt} = f(t, y)$ , and if its graph passes through the point  $(t_1, y_1)$  where  $y_1 = y(t_1)$ , then the differential equation says that the derivative  $\frac{dy}{dt}$  at  $t = t_1$  is given by the value  $f(t_1, y_1)$  [3, pg 34].<sup>7</sup> In other words the slope of the tangent line to the graph  $y(t)$  at any point  $(t_1, y_1)$  is the number  $f(t_1, y_1)$

The example shown below is the slope field of  $\frac{dy}{dt} = -2ty^2$ . To calculate this one chooses two coordinates, in this case the coordinates are  $(t, y)$ . Given values of  $t$  and  $y$  one finds the value of the slope at that particular point. So for  $(1, 1)$  the slope,  $\frac{dy}{dt}$  would be  $-2$ . (See Figure 0.1)

## Example of Slope Field

### Harmonic Oscillator

The goal is to establish a working mathematical model using differential equations that describes the motion of an object of mass  $m$ , suspended on a spring with spring constant  $k$ , and moving in a fluid that produces a restoring friction force proportional to speed,  $-c\frac{dx}{dt}$ .

Using Newton's law ( $F = mA$ ) the model becomes

$$m \frac{d^2x}{dt^2} = -c \frac{dx}{dt} - sx$$

where  $x$  is the displacement of the spring and  $t$  is time [4, pg 447].

### Derivation of the Model

-Graphics-

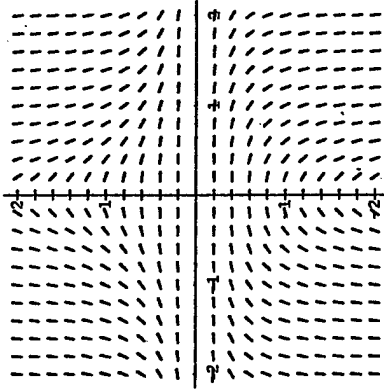


Figure 0.1: Slope Field of  $\frac{dy}{dt} = -2ty^2$

Newton's law ( $F = mA$ ) states that the total applied force equals the mass times acceleration that the force produces. Using this equation one obtains mass  $m$ , and acceleration which is the second derivative.

$$m \frac{d^2x}{dt^2} = F$$

Now to derive the force one needs to visualize all the forces acting. There are two forces acting that one is going to assume. They are the Shock Absorber Force and the Spring Force. Other forces can be added to the model, but the model tends to become more difficult. The spring force can be represented as  $-sx$ . When one stretches a string of length  $x$ , the restoring force acting on the displacement is negative. Likewise when the spring is compressed, this suggest that the displacement ( $x < 0$ ) is negative. Therefore the restoring force becomes positive.

The force of the shock absorber can be represented as,  $-c\frac{dx}{dt}$ . This force is also negative for the same reason concerning displacement. The researcher is using a derivative here because velocity as to be measured.



One may think of this equation ( $m \frac{d^2x}{dt^2} = -c \frac{dx}{dt} - sx$ ) as a model of a car when it goes over a bump. The constants  $m, c$ , and  $s$  are describing the design of the car for small periods of time. When the shock absorber wears out,  $c$  becomes smaller. When the car rides over a bump very fast, and the car has a good shock; then one expects the car to reach equilibrium at a reasonable amount of time. This differential equation will model all of these scenarios for the performance of a vehicle's shock absorber.

### Analyzing the Model

#### Phase Variable Trick:

The researcher first wants to analyze this model as a two-dimensional system by introducing a "phase variable",  $\frac{dx}{dt} = y$ .

Essentially the model,

$$\left( m \frac{d^2x}{dt^2} = -c \frac{dx}{dt} - sx \right)$$

can be rewrite as

$$m \frac{dy}{dt} = m \frac{d^2x}{dt^2} = -c \frac{dx}{dt} - sx = -cy - sx$$

because

$$\frac{dy}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2x}{dt^2}$$

If one combines these two equations one creates a system of first order equations,

$$\frac{dx}{dt} = y \quad \frac{dy}{dt} = \frac{-s}{m}x - \frac{c}{m}y$$

#### Linear Classification

This linear system is a variant of

$$\begin{aligned} \frac{dx}{dt} &= a_1x + a_2y \\ \frac{dy}{dt} &= b_1x + b_2y \end{aligned}$$

where

$$\begin{aligned} a_1 &= 0 & a_2 &= 1 \\ b_1 &= \frac{-s}{m} & b_2 &= \frac{-c}{m} \end{aligned}$$

This can be shown in matrix form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

or,

$$\dot{x} = Ax$$

$$\dot{x} = ax + by \quad (0.1)$$

$$\dot{y} = a \dot{x} + b \dot{y} \quad (0.2)$$

$$\dot{y} = a \dot{x} + b(cx + dy) \quad (0.3)$$

$$\dot{y} = a \dot{x} + bcx + bdy \quad (0.4)$$

$$d \dot{x} = adx + bdy \quad (0.5)$$

By subtracting equations 0.4 and 0.5 one gets,

$$\dot{y} - d \dot{x} = a \dot{x} + (bc - ad)x$$

$$\dot{y} - (a + d) \dot{x} + (ad - bc)x = 0$$

One can say that  $(a + d)$  is the  $\text{tra}$  of the matrix  $A$  and  $(ad - bc)$  is the  $\text{det}$  of the matrix  $A$ .

Therefore the characteristic equation is

$$\lambda^2 - \text{tra} \lambda + \text{det} A = 0 \quad (0.6)$$

and

$$\lambda = \frac{\text{tra} \pm \sqrt{(\text{tra})^2 - 4 \text{det} A}}{2}$$

The discriminant is equal to  $(trA)^2 - 4 \det A$ .

The characteristic equation obtained using linear differential equations is the same characteristic equation obtained from

$$\det \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix} = 0$$

$$(a-\lambda)(d-\lambda) - bc = 0 \quad (0.7)$$

$$ad - a\lambda - d\lambda + \lambda^2 = 0 \quad (0.8)$$

$$\lambda^2 - \lambda(a+d) + ad - bc = 0 \quad (0.9)$$

As one can see equations 0.7 and 0.9 are essentially the same equation. This shows the direct relation between linear systems of differential equations and linear algebra methods.

The equilibrium point of this shock absorber model is at the origin (0,0). This is obvious because the force of the shock is either going to stop (attract) or it will continue infinitely (repell). When modeling the shock absorber one knows that the force could never repel from the fixed point because this implies that the car would never stop bouncing after it hits a bump, but to gain a theoretical picture of the model and complete understanding this unrealistic behavior must be shown. Complete understanding of this system will lead to the basic idea "nonlinear phenomena locally look linear [4, pg 441]."

#### Symbolic Exponential Analysis:

This method of analysis suggest one assumes the solution to the model is exponential. Let's first consider the second-order differential equation which is exactly like the harmonic oscillator model with different variables.

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = 0$$

Now assume the solution is of exponential form  $x(t) = e^{rt}$ .

$$x = e^{rt} \quad cx = ce^{rt}$$

$$\frac{dx}{dt} = re^{rt} \quad b \frac{dx}{dt} = br e^{rt}$$

$$\frac{d^2x}{dt^2} = r^2 e^{rt} \quad a \frac{d^2x}{dt^2} = ar^2 e^{rt}$$

Therefore,

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = (ar^2 + br + c)e^{rt}$$

There is no time when  $e^{rt} = 0$ , so one must set  $(ar^2 + br + c) = 0$  to satisfy the equation.

Now one has reduced the differential equation into a algebra problem. This algebraic equation  $(ar^2 + br + c) = 0$  is called the *characteristic equation* of the differential equation and the roots of the characteristic equation are called *characteristic roots* [4, pg 450].

Then one summarized the basic solutions of second order differential equations which models the shock absorber in the following theorem.

#### Theorem

The following procedure gives a pair of linear independent basic solutions to the differential equation

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = 0$$

in the case when  $a \neq 0$ .

Find the solutions  $r_1$  and  $r_2$  of the characteristic equation  $ar^2 + br + c = 0$

1. If the roots is repeated,  $r_1 = r_2$ , then  $x_1(t) = e^{r_1 t}$  and  $x_2(t) = te^{r_1 t}$
2. If the roots are real and distinct, then  $x_1(t) = e^{r_1 t}$  and  $x_2(t) = e^{r_2 t}$
3. If the roots are a complex conjugate pair  $r_1 = \alpha + i\beta$  and  $r_2 = \alpha - i\beta$ , then  $x_1(t) = e^{\alpha t} \cos(\beta t)$  and  $x_2(t) = e^{\alpha t} \sin(\beta t)$

Every solution to this differential equation can be written as  $x(t) = k_1 x_1(t) + k_2 x_2(t)$  for suitable choices of constants  $k_1$  and  $k_2$ . The researcher wrote a Mathematica Program to understand the dynamics of the solution graphically for different initial conditions  $x(0) = x_0$ ,  $y(0) = \frac{dx}{dt}(0) = y_0$ , and different values of  $a, b, c$  (appendix B).

To summarize the study of the dynamics of second order differential equations the following various possibilities can occur in the dynamics.

#### Stability Analysis

Now using this characteristic equation for different values of  $a, b, c$  one can visualize what the model is doing. One wants to find out when the solution of the differential equation attracts or repels from the equilibrium point. The equilibrium points being the characteristic roots of the characteristic equation.

There are eight cases that can possibly occur and must be checked. The reference pictures for these eight cases can be seen in Appendix A.

Using Mathematica one is able to check each of these cases to determine whether the solution attracted or repelled (Table 0.1). Each particular case can be proven analytically also. Here are examples of two cases.

Example: Roots are distinct and negative

Assume the roots are  $r_1 = -1, r_2 = -2$ .

This suggest that the characteristic equation is

$$(r+1)(r+2) = 0$$

$$r^2 + 3r + 2 = 0$$

Therefore if one takes the limit of  $y = c_1 e^{-t} + c_2 e^{-2t}$  as  $t$  approaches infinity one can see that the solution attracts to the equilibrium value.

Example: Roots are distinct and positive

Assume the roots are  $r_1 = 1, r_2 = 2$ .

This suggest that the characteristic equation is

$$(r-1)^2 = 0$$

$$r^2 - 2r + 1 = 0$$

Therefore if one takes the limit of  $y = c_1 e^t + c_2 e^{2t}$  as  $t$  approaches infinity one can see that the solution repels to the equilibrium value.

All of these cases can be shown graphically. Figure 0.2 shows the region where each case is stable. Hence one has predicted the long term behavior of the harmonic oscillator (homogeneous case) [1, pg 470].

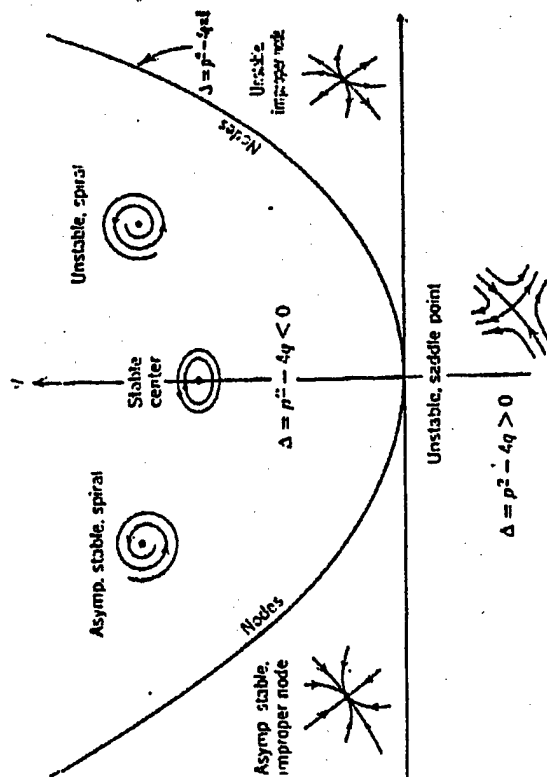


Figure 0.2: Stability Diagram

Possible Cases	Phase Portrait	App. C
Roots are distinct and negative	Attracting Proper Node	A.1
Roots are repeated and negative	Attracting Improper Node	A.2
Roots are complex but negative real part	Attracting Spiral	A.3
Roots are complex but with zero real part	Stable Center	A.4
Roots are distinct and positive	Repelling Proper Node	A.5
Roots are repeated and positive	Repelling Degenerate Node	A.6
Roots are complex but positive real part	Repelling Spiral	A.7
Roots are real but opposite of signs	Saddle	A.8

Table 0.1: Shock Absorber's possible cases

## Appendix A

### Shock Absorber – Reference Pictures

#### A.1 Reference Picture 1 — Shock Absorber

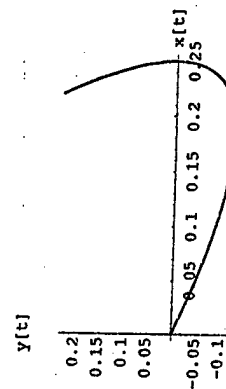


Figure A.1: Shock Absorber — roots are distinct and negative

## A.2 Reference Picture 2 — Shock Absorber

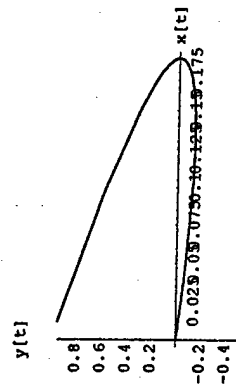


Figure A.2: Shock Absorber — roots are repeated and negative

## A.3 Reference Picture 3 — Shock Absorber

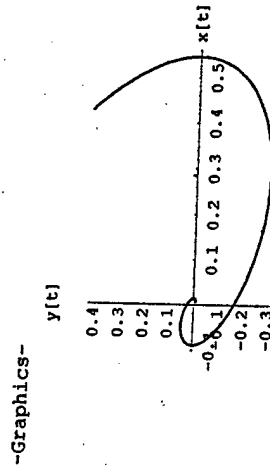


Figure A.3: Shock Absorber — roots are complex but negative real part

#### A.4 Reference Picture 4 — Shock Absorber

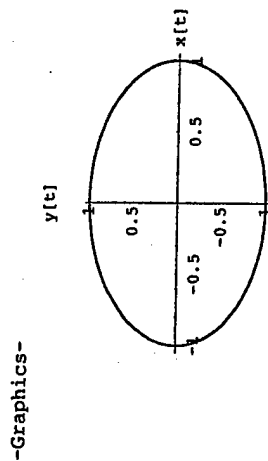


Figure A.4: Shock Absorber — roots are complex but zero real part

#### A.7 Reference Picture 7 — Shock Absorber

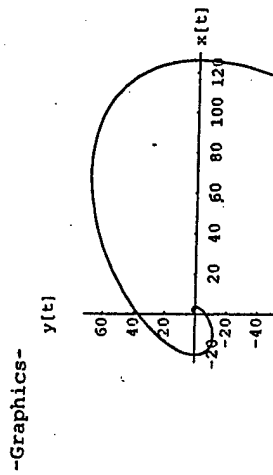


Figure A.7: Shock Absorber — roots are complex but positive real part

## Appendix B

### Mathematica Program

#### B.1 Harmonic Oscillator Program

This program was designed to graphically show the Harmonic Oscillator Model (Homogeneous).

```
Clear[a,b,c,t]
```

```
a = 1;
```

```
b = -5;
```

```
c = 6;
```

```
If[b^2-4 a c == 0,
```

```
  r = -b/(2 a);
```

```
  x1[t_] := Exp[r t];
```

```
  x2[t_] := t Exp[r t];
```

```
],
```

```
  If[b^2-4 a c < 0,
```

```
    w = Sqrt[4 a c - b^2]/(2 a);
```

```
    r = -b/(2 a);
```

```
    x1[t_] := Exp[r t] Cos[w t];
```

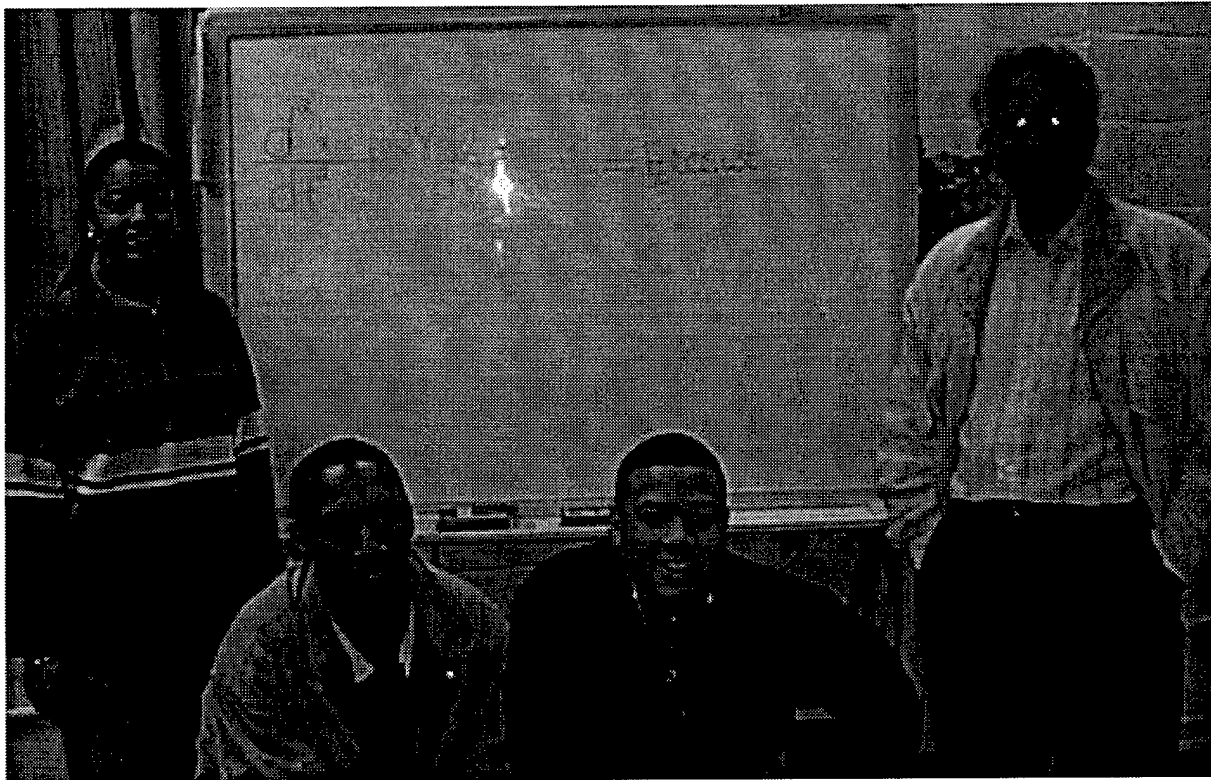
```
    x2[t_] := Exp[r t] Sin[w t];
```

```
  r1 = (-b + Sqrt[b^2-4 a c])/(2 a);
  r2 = (-b - Sqrt[b^2-4 a c])/(2 a);
  t = 0;
  m = {{x1[t],x2[t]},{y1[t],y2[t]}};
  TableForm[m];
  xi = 0.0;
  yi = 1.0;
  Coefs = LinearSolve[m, {xi,yi}];
  k1 = Coefs[[1]]
  k2 = Coefs[[2]]
  Clear[t]
  x[t];
  Plot[x[t],{t,0,3}]
  ParametricPlot[x[t],y[t],{t,0,10}]
```

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# *Physics*



*Team Mentor: Dr. L. Choudhury*

*Team Members: Santiel Creekmore  
Katrina Godwin  
Arthur Fenner*



# Power Dissipation of a Damped Harmonic Oscillator Under the Influence of a Periodic Driving Force

Power Dissipation of a Damped Harmonic Oscillator Under the Influence of a Periodic Force

## Abstract:

In this work we carry out mathematical formulation of the average power dissipation of a damped harmonic oscillator under the influence of a periodic driving force. We set up the equation of motion of a mass  $m$ , according to the second law of Newton, tied to a spring moving under the simultaneous action of the air resistance and a periodic force. It leads to an inhomogeneous second order linear differential equation. We solve the equation rigorously. Regulating the parameters, we can make the homogeneous part of the solution dampen rapidly. We then calculate the average power of the system over the period of the driving force. We get:

$$P_{av} = \frac{F_0^2}{4\gamma m} \frac{4\gamma^2 \omega^2}{(\omega^2 - \omega_0^2)^2 + 4\gamma^2 \omega^2}$$

Where  $\omega_0$  is the intrinsic angular frequency of the spring,  $\omega$  is the angular frequency of the driving force,  $\gamma$  is the damping parameter, and  $F_0$  is the magnitude of the driving force. We finally develop a Mathematical program to plot a three dimensional diagram of the average power depending on  $\omega$  and  $\gamma$ . We explicitly demonstrate how the power increases as the driving frequency approaches the resonating frequency of the spring.

## Introduction:

Harmonic oscillators play an important role in physical processes. In its simplest version, we create it by tying a mass to one end of a spring and keeping the other end fixed and letting it slide on a horizontal perfectly smooth plane as shown in Figure 1. We drag the mass  $m$ :

PHYSICS TEAM:  
SANTEL J. CREEKMORE  
ARTHUR FENNER  
KATRINA Y. GODWIN

MENTOR:  
Dr. A. L. Choudhury  
Department of Physics  
Elizabeth City State University

power dissipation under the assumption that the system damps rapidly. In section 4, we first develop Mathematica program to graph the amplitude of the wave. We also write a program to generate a three dimensional graph of the average power as a function of two parameters. We demonstrate clearly the resonating shape of the average power. In section 5, we make some concluding remarks.

## 2. Equation of Motion for Harmonic Oscillator

If a mass attached to a spring is pulled to a length  $x_0$  and then let go, the force it is subjected to is given by

$$(2.1) \quad F = F_s + F_r$$

where

$$(2.2) \quad F_s = -kx \quad \text{and} \quad F_r = bv_x$$

In Eq.(2.2)  $k$  is the spring constant, and in (2.3)  $b$  is the proportionality constant of the air-resistance. The equation of motion becomes

$$(2.3) \quad m \frac{d^2x}{dt^2} = F.$$

We can change the whole equation into the following form:

$$(2.4) \quad \frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = 0$$

where we have set

$$(2.5) \quad \gamma = \frac{b}{2m} \quad \text{and} \quad \omega_0^2 = \frac{k}{m}.$$

If we set a test solution  $x = e^{pt}$ , the auxiliary equation turns out to be

$$(2.6) \quad p^2 + 2\gamma p + \omega_0^2 = 0.$$

The solution of this quadratic equation becomes

$$(2.7) \quad p = -\gamma \pm i\omega_1$$

where

$$(2.8) \quad \omega_1^2 = \omega_0^2 - \gamma^2.$$

to a position  $x_0$  and then let the mass go. The mass  $m$  then generates a simple harmonic motion. It moves back and forth about a fixed point. Under the restraining action of air resistance, it generates a damped motion.

In nature we encounter such a vibration very frequently. Every molecule of a crystal executes simple harmonic motion. When a wheel of a watch oscillates, it generally executes simple harmonic motion. Similar motion can also be found in a pendulum.

We can then subject that mass to a periodic force. The motion of mass under this periodic force is very interesting for engineers and physicists.

In this work we study rigorously the basic idea involving this motion. We set up the fundamental equation for the position of the mass, starting from Newton's second law of motion.

In section 2, we set up that equation and convert it into a second order differential equation of degree one. This equation turns out to be an inhomogeneous differential equation. We then solve this equation rigorously.

We use this solution to find the power of the system in section 3. Then we find the average

We want to generate a damped oscillatory motion, we choose

$$(2.9) \quad \omega_0^2 < \gamma^2$$

The final solution turns out to be

$$(2.10) \quad x = A e^{\gamma t} \cos(\omega_1 t + \theta)$$

where  $A$  and  $\theta$  are two arbitrary constants.

The energy of the damped oscillator is given by

$$E = \frac{m\dot{x}^2}{2} + \frac{kx^2}{2}$$

$$= \frac{m\Delta^2 e^{2\gamma t}}{2} [\gamma^2 \cos^2(\omega_1 t + \theta) + \omega_1^2 \sin^2(\omega_1 t + \theta) + 2\gamma\omega_1 \cos(\omega_1 t + \theta)] + \frac{k\Delta^2 e^{2\gamma t}}{2} [\cos^2(\omega_1 t + \theta)]$$

In the limit  $\gamma \rightarrow 0$ ,  $\omega_0 \gg \gamma$ , we get

$$(2.12) \quad E = k\Delta^2 e^{2\gamma t}$$

In that limit the rate of change of  $\ln E$  becomes:

$$(2.13) \quad \frac{1}{E} \frac{dE}{dt} = -2\gamma$$

In addition if we now add a periodic force

$$(2.14) \quad F_p = F_0 \cos \omega t$$

the equation of motion changes into the following form

$$(2.15) \quad \frac{d^2 x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = F_0 \cos \omega t$$

To solve the equation, we introduce an extra equation

$$(2.16) \quad \frac{d^2 y}{dt^2} + 2\gamma \frac{dy}{dt} + \omega_0^2 y = F_0 \sin \omega t$$

Introducing an amplitude  $z$  defined by the relation

$$(2.17) \quad z = x + iy$$

we can combine Eqs. (2.13) and (2.14) into the form

$$(2.18) \quad \frac{d^2 z}{dt^2} + 2\gamma \frac{dz}{dt} + \omega_0^2 z = \frac{F_0 e^{i\omega t}}{m}$$

The solution of Eq(2.17) can be written as

$$(2.19) \quad z = z_h + z_p$$

where  $z_h$  satisfies the equation

$$\frac{d^2 Z_H}{dt^2} + 2\gamma \frac{dZ_H}{dt} + \omega_0^2 Z_H = 0 \quad (2.20)$$

and  $Z_p$  the particular solution, satisfies

$$\frac{d^2 Z_p}{dt^2} + 2\gamma \frac{dZ_p}{dt} + \omega_0^2 Z_p = \frac{F_0}{m} e^{i\omega t} \quad (2.21)$$

The solution of Eq.(2.20) has exactly the same form as of Eq.(2.10), that is

$$Z_H = A e^{-i\omega t} \cos(\omega_0 t + \theta) \quad (2.22)$$

To obtain the particular solution, we set the test form of  $Z_p$  as

$$Z_p = B e^{i\omega t} \quad (2.23)$$

Substituting this form in Eq.(2.21), we find

$$B = \frac{F_0/m}{\omega_0^2 - \omega^2 + i2\gamma\omega} \quad (2.24)$$

Rationalizing the denominator we find

$$Z_p = \frac{F_0}{m} e^{i\omega t} \left[ \frac{\omega_0^2 - \omega^2 - i2\gamma\omega}{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2} \right] \quad (2.25)$$

The  $X_p$  comes out to be

$$X_p = R_C Z_p = R_C Z_H + R_C Z_p$$

$$X_p = A e^{-i\omega t} \cos(\omega_0 t + \theta) + \frac{F_0}{m} \cos \left( \omega t - \tan^{-1} \frac{2\gamma\omega}{\omega_0^2 - \omega^2} \right) \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \quad (2.26)$$

where we have chosen A and theta to be real.

Introducing

$$\beta = \frac{\pi}{2} - \bar{\beta} \quad (2.27)$$

where

$$\bar{\beta} = \tan^{-1} \frac{2\gamma\omega}{\omega_0^2 - \omega^2} \quad (2.28)$$

We can write

$$X = A e^{-i\omega t} \cos(\omega_0 t + \theta) + \frac{F_0}{m} \frac{\sin(\omega t + \beta)}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \quad (2.29)$$

The first term, called the transient state disappears after a sufficiently long time. The second term is called the steady state.

### 3. Average Power

In a steady state, the power is given by the expression:

$$P = \text{Driving Force} \times \text{Speed} = F_p \cdot V_x$$

$$P = D(\omega) \left[ (\cos^2 \omega t) \cos \beta - \frac{1}{2} \sin 2\omega t (\sin \beta) \right] \quad (3.1)$$

where we have set

$$D(\omega) = \frac{F_0^2 \omega}{m \sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \quad (3.2)$$

We define the average power,  $P_{av}$  as

$$P_{av} = \frac{1}{T} \int_0^T P dt \quad (3.3)$$

whose  $T = 2\pi/\omega$ , which is the period of the driving force.

We get

$$P_{av} = \frac{\omega}{2\pi} D(\omega) \left[ (\cos \beta) I_1 - \frac{1}{2} (\sin \beta) I_2 \right] \quad (3.4)$$

whose

$$I_1 = \int_0^{\frac{2\pi}{\omega}} \cos^2 \omega t dt = \frac{\pi}{\omega}$$

$$I_2 = \int_0^{\frac{2\pi}{\omega}} \sin 2\omega t dt = 0$$

Then we get

$$P_{av} = \frac{F_0^2}{4\gamma m} \frac{4\gamma^2 \omega^2}{(\omega_0^2 - \omega^2)^2 + 4\gamma^2 \omega^2} \quad (3.5, 3.6)$$

(3.7)

Finally we plot the average power  $P_{av}$  from different perspective. The program runs as follows:

```
F[w_,g_] = A(g w^3)/((w^2-w0^2)^2 + 4g^2 w^4);
A=1; w0=3;
```

```
Plot 3D->[F[w,g],{w,0.5},{g,.001,3}, Plot Range->{0,1}
ViewPoint->{a,b,c}, Axes Label->{w,g,Pav}]
```

For ViewPoint-> default we get the outcome in Fig. 6.

For a=5,b=0,c=0, we get the sideview as shown in Fig7. Fig8. gives the backside view. We notice that the resonance is prominent at  $w=w_0$ .

#### 5. Concluding Remarks

We have taken a simple vibrating system and demonstrated how to solve the problem using fundamental methods of differential equation. The solution obtained has been plotted using Mathematica Program. We found out that the driving force dominates the power dissipation rapidly depending on the value of the damping factor gamma. It will be very interesting to study the effect on such resonance with a driving force of delta function type we intend to do it next year.

#### Acknowledgements

First and foremost we would like to thank Dr. Hayden and the entire Faculty and staff of the ONR program for all of the help, facilities, and encouragement provided for our research. We would especially like to thank Dr. Choudhury for all of the time, patience, and knowledge that he has shared with us.

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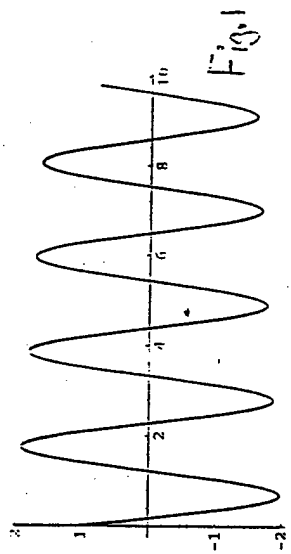


Fig. 1

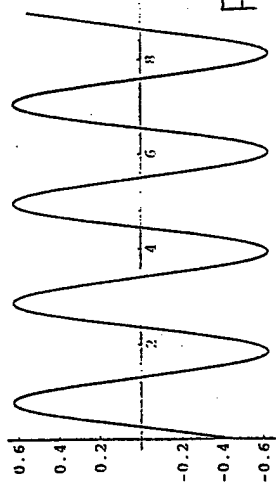


Fig. 2

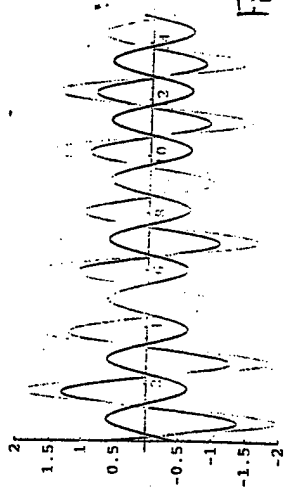


Fig. 3

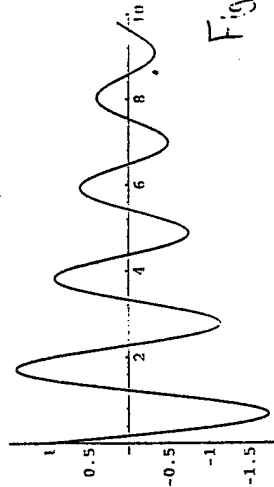


Fig. 4

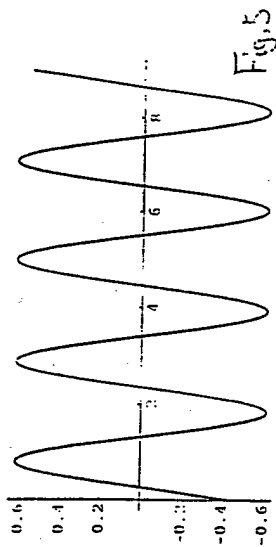


Fig. 5

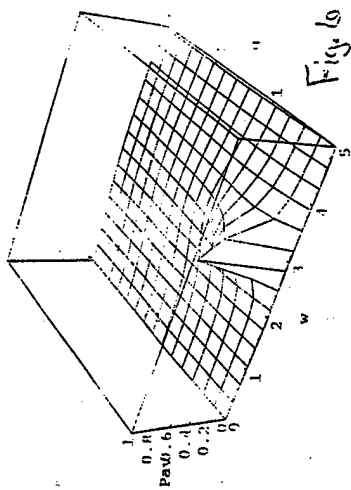


Fig. 6

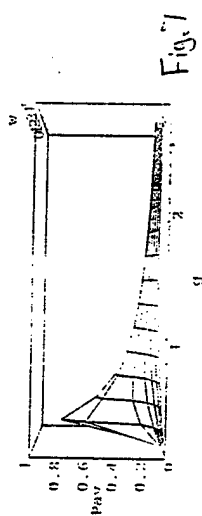


Fig. 7

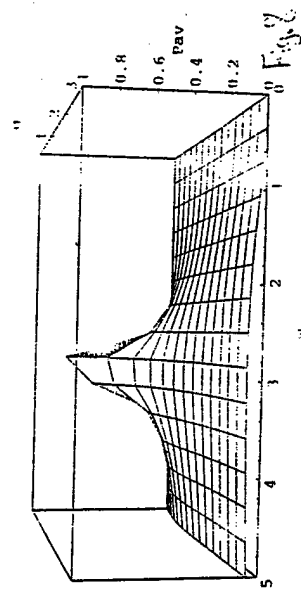


Fig. 8

# *Multimedia Authoring*



*Team Mentor: Amie Aydlett*



*Team Members: Je'aime Powell, Donald Charity, and Jonathan Williams*

# Multimedia Team Final Research Report

The goals of the 1997-1998 Multimedia Team was to learn the software from Macromedia entitled Authorware, and after mastering its concepts, organize, create, and produce a presentation that would educate others on what and how Authorware does.

During the beginning of the research, the members of the team first began to educate and teach themselves the application software. All members of the research group were given copies of the Authorware Working Model Guide Book. Each member began using the tutorial compact disc enclosed with the guide. During the first three lessons, all members worked together on learning the basic principals. The next six lessons were then divided up evenly upon the three members of the team. Upon each consecutive meeting thereof, each member presented their learnings, which was saved on disk, to each of the other members and taught their assigned lesson to them.

Also during this time, the group was taken to the United States Coast guard base in Elizabeth City, North Carolina where the personnel showed the group some of their outstanding educational presentations and beneficial creations. They also stored one of their files on a disk for the group to use to give them additional assistance in creating their projects.

During the next few weeks, members of the team were assigned various tasks which included the main presentation, which was split into two parts, the team homepage, the research report, and the team articles reviews.

Developing the presentation portion of the research at one point was a problem. This was because of the limitations of the version of

Authorware that each researcher was equipped with. After our advisor provided us with a full upgrade version of the program, the presentation was able to be completed in an easy fashion, even though the transition of the old files and incorporating them into the newer one was a bit of a hindrance.

The creation of the homepage was a relatively simple project as the team's knowledge of HTML assisted greatly to developing it. The articles were read and summaries were provided in a matter of days. The typographical errors were corrected and have been resubmitted in their final form.

The final research report was produced after the completion of all the team's research. It extensively covered all work done by the team. The team's final assignment which has been completed but not presented is their presentation to the students, faculty, and guests entitled "Authorware: From Us, To You" which is a formal presentation of the team's work on Authorware. At this session, each student will be provided with a disk containing a shocked version of the team's presentation which can be played on any Internet browser including Netscape Navigator and Microsoft Internet Explorer. This concluded the Multimedia's Team research and assignments that were to be completed during the 1997-1998 ECSU school year.



April 7, 1998

pg. 34-36

## The Web Is In Motion - On-Line Multimedia Is Limited, But Not For Long

by George Kepper

Multimedia Producer

In this article reviewed by the Multimedia Team, the author describes in a very in detailed discussion the progress that the multimedia is making with the Internet. It overallly attempts to display the progress the Internet has made from a static web page to a page full of multimedia content including animation, audio, and video.

At this time, software and hardware are being developed in order to accomplish this long term goal. Bandwidth limitation was a problem at once, but now most network users are at ISDN, T1, and T3 connection speeds. And home users now have the 56Kbps option available. Arrangements are now being made with television stations, satellite operators, and various companies in order to aid this task.

Multimedia applications minimally require an Internet connection of 14.4Kbps. In addition, users must have the proper hardware and software to support the rich multimedia content such as particular video and audio cards, and a capable operating system such as Windows 95 and above or Macintosh operating system 7.5.3 or above. They also need suitable browsers such as Netscape Navigator 2.0 and above or Microsoft Internet Explorer (MSIE) 3.0 and above. Both browsers are now well

prepared for such content with version 4.05 for Netscape, which was released on April 2, and version 5.02 for MSIE, which was released on March 29. Most of these browsers use helper application to play such multimedia content. One such application is RealAudio, which now goes by the name of RealPlayer. This application is capable of playing streaming Internet audio and video over ordinary phones and cable lines at supporting speeds as low as 14.4Kbps. Now even users who can not afford such costly modems can enjoy the same benefits as others. Another such type of helper application is Macromedia Shockwave, which like the RealPlayer, allows the user to listen, play, and rewind without waiting for downloads. This type of content only gets the header or the first part of the file and downloads the rest while the earlier portion is playing using the user's RAM and cache memory. This breakthrough makes it possible to have live video and audio conversation with another users or watch a live telecast from those providers that use it such as CNN and C-SPAN.

One type of animation that does not rely on helper applications is he moving GIF. This type, also called GIF89, allows the multi-image frames to be stored as a single GIF file. This technique is much simpler than using the push and pull technique of CGI to gather information. This form of animation is becoming more and more popular because it does not require that the user have anything other than the standard capable browser.

The final topic Kepper covers is Java. Java is a type of animation which incorporates the technology for real time animation as the data can be delivered live such as with stock tickers and global news. Java applets can be easily displayed through a Java-enabled browser such as those previously stated. This language is advancing even

more with Javascript which is more HTML based. This can provide information using cookies and even a fading color background upon the entrance of a web document.

## **A Tale of the Tape**

### **When mining vaults yields unpleasant surprises.**

Dan Daley

Television has become the heaven of old nostalgic video clips and shows. The only problem with this is the fact that magnetic tapes and films become less pleasing with every use. Deterioration from usage or storage has some terrible effects on the quality of a tape. Historical or periodical tapes can become unusable due to the quality losses. Heat and Humidity are responsible for most of the problems with archived analog tape. A climate controlled vault-preferably with Fahrenheit temperature in the 60s and a humidity level below 50 percent is ideal, but historically, most analog tape has not been stored in these conditions. Bill Creed, video editor of a documentary about Lyndon Johnson that recently aired on England's Channel Four and on The History Channel in the United States, says although the staff at the National Archives in Washington did their best to make hundreds of hours of LBJ recordings audible, they could go only so far. Creed said they had to choose what was audible from the notes of LBJ's secretary from a Dictaphone rather than best. Things that were seen as very important were sent out to be improved using EQ and digital enhancement. Lou Gonzales, an engineer and owner of Quad Recording in New York City, dealt several years ago with neglected recordings for a major project. Some John Lennon recordings he had received had been stored for years in a humid unairconditioned environment. In cases such as this, the binder (the glue that holds the oxide tape formulation to the backing material) turns into a molasses like substance and sometimes takes the program material with it. Gonzales devised a technique in which a sponge-like cloth positioned before the

playback heads wiped the tape clean of excess goop. It was a painstaking process with many starts and stops but it was the only way to retrieve the music.

Another solution for fixing the binder is to bake the tape, a common technique in which older analog tapes are placed in a convection oven for 24 hours and heated at Fahrenheit temperature in the low 100s. The process will resolidify the binder, but "what you have to do is make a copy of that tape very quickly," Gonzales says. "Otherwise, the baking process will unleash other degradation and you could lose the tape altogether."

Several companies, including Sonic Solutions and CEDAR Audio, manufacture hard-disk-based systems that use electronic recomposition to eliminate or reduce certain types of problems often found on older and improperly stored analog and digital tapes. Functions such as declicking, decrackling and dehisssing have become major elements of restoring analog audio, particularly vinyl records. The disk-based systems analyze pops, clicks and scratches, and use algorithms that interpolate and replace missing data from program material.

Although these techniques help some what in retrieving archival information of the past many historical recordings are still lost forever. DVD technology will hopefully stop the loss of records from happening again. This new technology is laser driven as well as a lot easier to store. Hopefully these new recordings will last for all time.



## Executive Animation

Author: Claudyne Wilder

Video & Multimedia Producer, April 1997, p. 37

Have you recently given a presentation created by a colleague or friend which looks great but just isn't in sync with what you really wanted, or have you personally ever done a presentation which you later found took away from the speaker more than helped him/her? Perhaps you didn't follow the ten essential rules of presentation creation. These rules were created by the Author of the story 'Executive Animation' in the April edition of Video and Multimedia Producer, Claudyne Wilder. She had recently worked with an executive who had just received some screens of presentation created for his big meeting. He wanted help with his presentation style and dramatic delivery. Along the course of the session it was found that the presentation itself was at fault in making him feel out of place. The author then had to revamp the presentation causing the company to loose money and development time. The author explains the down time wasn't as much a problem but the fact that it all could have been avoided by having the presentation creator to be included. After this event she created some basic rules for the problem to not happen again. The basic rules are:

1. **Match the styles of the presenter and the presentation.** - The first thing you need is the presenters presenting style. Is his or her style conservative or moderate? Enthusiastic? Animated? You don't want to put together a bells and

From the ONR-ASSERT NDET Research Lab	Presenting The Multimedia Group	Members Hot Spots: <input type="checkbox"/> Pictures <input type="radio"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Picture Name mp11 Hot
←	Same as last (each member does fancy page for them self)	→	Screen Purpose: _____ _____ _____
Authorware  (Macromedia)	(Authorware shows off demo)	Authorware Basics 	The Queue Call hot spots leading to corresponding page)
_____	_____	_____	_____
_____	(pages for navigation buttons)	_____	The  Ew

whistles talk with sound, animation and lots of wild movement when the person will be sitting reading a script.

2. **Avoid Scripts.** - First, a script anchors the presenter to one spot and can hamper his or her ability to sound animated. Second, scripts can keep presenters from establishing rapport with the audience. Instead of scripts, designing the screens as cue cards is suggested.

3. **Involve the presenter early in the process.** - If you involve the presenter early the presenter will both be more comfortable and animated in the presentation when giving it.

4. **Make the visuals self explanatory.** - Unusual visuals will loose the audience.

5. **Use headings to cue the speaker.** - The heading should contain the screens main point, serving as a cue to the speaker as soon as the screen appears. Get to know the presenter, if possible, and talk to him or her. - This adds the character of the speaker to the presentation.

Remember that you have the power to make a presenter truly compelling or truly boring. Take the time and energy to make your presenter look outstanding.

4/ 3/ 98  
pp. 47-52

## Breaking Through Into VR

By: Louis M. Brill

Multimedia Producer

In conclusion, the future of virtual reality is here today, applications of VR have been adapted in commercial, scientific, and industrial communities. In today's society people do not just look at media anymore they want to interact with it, this is one of the advantages that virtual reality presents to the user. This calls for more training for the users. Because of the users of multimedia desire to touch and move things, virtual reality and multimedia will converge together towards the future.

In the article reviewed by the Multimedia Team, "The Whole New World" the author compares the advances of multimedia to the advances of virtual reality (VR). Multimedia businesses are now using the same equipment virtual reality businesses are using to create their presentations. Also multimedia developers are now becoming more familiar with virtual reality applications and incorporating them into their own developments. The user buying the multimedia package is gaining more than multimedia but they are also gaining a more interactive package as an virtual reality application. With the equipment multimedia developers are using today; they will be able to develop their own VR experience. Virtual reality creates the effect of objects being there but not actually. Combined with multimedia, VR gives an extra edge to the user, it draws them closer and causes them to become more involved in the presentation. Even though multimedia and VR can be incorporated together presenting the same features, they also differ in the same instance. In multimedia the information is presented by text, graphics, and sound. Virtual reality on the other hand is presented in 3-D format, shapes, and sound using no traditional tools to guide the user through the application. Also in virtual reality all the projects completed are strictly experimental. In today's VR development market, the developers are asking how best to use the technology. In making a transition to VR, comes the question of how will they gain funding for the upgrade and how will they train the developers to use the new equipment.

# *Java Script*



*Team Mentor: Dr. R. Kalkarnie*

*Team Members: Angela Mizelle  
Lakisha Mundon  
Sheri Joyner  
Micheal Boisson  
Joseph Gale*

## HTML/JavaScript Team

### Abstract

Student HTML/JavaScript researchers learn to produce documents in Hyper Text Markup Language (HTML), the language used to create web pages. Specifically, they will explore the following topics:

Introduction to the World Wide Web; the structure HTML pages - tags, head, title, body, end tags; headings - bold, underline, italics; paragraphs; lists; links; images; tables; background and text color, frames; windows; introduction to JavaScript

The student researchers will explore the above topics in a step by step manner. At the end of their training, they will be able to update and maintain ONR/NERT web pages.

### Implementation

The training started with HTML and culminated with an introduction to JavaScript. Some members of the team did not have much prior experience with programming. Hence, during the semester, their training involved with a cycle of a session of lecture exploring some HTML topics and a session on hands on experience with the computer exploring the concepts learnt. Most of the second semester was used for learning the concepts and independently carrying out the hands on experience and also going through the HTML tutorials on the Internet. In their training, they explored specifically the following topics:

The world Wide Web; web browsers; Internet domain extensions; Structure of HTML pages - tags, end tags, head, title, body; A simple web page; Paragraphs; Text formatting - headings, horizontal rule, bold, underline, line breaks, pre-formatted text, the dreaded blink; Lists - ordered lists, unordered lists, definition lists, menu lists, directory lists, nested lists; Links - linking local pages using relative and absolute path names, links to documents on the Web, links to specific places within the same document; Tables - creating basic tables: rows, cells, and captions, table and cell alignments; Images - inline images, text and image alignments, images and links; Changing the background and text color; Frames; Windows; Introduction to JavaScript - What is JavaScript; JavaScript and HTML; Uses of JavaScript - including dynamic information, validating forms, and making the web pages interactive; A simple JavaScript program; Objects and functions; A JavaScript program to print tables

```
<!--Presented by Angela Mizelle
On this page the following items are being presented
1. The definition of frames
2. How frames are used
3. An example of frames
-->
```

### Frames

When using frames you divide Web pages into multiple, scrollable regions. You can present information in a flexible and useful fashion. Each region has different features such as:

- It can be given an individual URL, so it can load information independent of the other frames on the page;
- It can be given a NAME, allowing it to be targeted by other URLs;
- It can be resized dynamically if the user changes the window's size.

A Frame document has a basic structure, except the body container is replaced by a frameset container which describes the sub-HTML documents, or frames, that will make up a page. Tags that would normally be placed in the body container can not be placed in the frameset tag, or the frameset will be ignored. The frameset tag has a matching end tag, and within the frameset you can only have nested frameset tags, frame tags, or the noframes tag.

Targeting windows allows the document writer to assign names to specific windows, and documents to always appear in the window bearing the matching name.

A name is assigned to a window in several ways such as:

- A document can be sent with the optional HTTP header
- A document can be accessed from a targeted link. In this case there is actual HTML which assigns a target window name to a link.
- A window created within a frameset can be named using the NAME attribute to the frame tag.

Targeting with HTML is accomplished by means of the target attribute. This attribute can be added to a variety of HTML tags to target the links referred to by the tag. The attribute is of the form:

```
TARGET="window_name"
```

An example of frames is presented below.

```
<html>
<head>
<title>Angela3.html</title>
</head>
```

```
<frameset cols="25%,**" frameborder="no" border="0" framespacing="0">
<frame src="angela1.html" scrolling="yes" name="left" frameborder="no" border="0">
```

```
<frameset rows="25%,**" frameborder="no" border="0" framespacing="0">
<frame src="angela.html" scrolling="yes" name="top" frameborder="no" border="0">
<frame src="angela2.html" scrolling="yes" name="right" frameborder="no" border="0">
</frameset>
```

```
</html>
```

The outcome of this example of frames would be three frames presented on one page.

## LINKS

A link is used to retrieve the location of a file.  
The link tag appears within the head section of a document.  
Links can be used for:

1. Links to other pages  
(link to profile from resume4 and back)
2. Links to other parts of the current page  
(link to a particular section of the same page and back to top)
3. Links to images and pictures  
(link to PIC.HTML)
4. Links to soundfiles

Links consist of a tag, a parameter, a the value. The code for the link is put wherever you want the link to show up. The link has a symbol, a section of text or a picture that a viewer can select (click on) in a web page to activate the link. The value is the address of the site you are making the link to. If you are using a link to another section in the same page, then you must put an Anchor at the top of the section you want to make the link to.

## ANCHORS

Anchors are used in links to other areas of the same page only. There are two parts to an anchor. One part is specifying the area that will be linked to, and the other is the link itself.

(The anchor tag defines either a link or an anchor in a document.)  
The anchor tag must contain an HREF attribute. The link must use a # (go to jot). The code for the link to another section is also put wherever you want the link to show up.

## TABLES

Tables are similar to ordered and unordered lists. Tables are simple because they only use four tags:

1. Table Heading <TH>
2. Table Row <TR>
3. Table Columns <TC>
4. Table Data <TD>

The table heading is basically the same as the regular heading command. The table row resembles the list item in an ordered list. Listed horizontally. Table columns are listed vertically. Table data is the text inserted in the rows and columns.

An example of Links, Anchors, and Tables are below.  
-RESUMES.HTML  
-TABLES.HTML

```
<! JOSEPH GALE
IN THIS RESUME WE INCORPORATED:
1. LINKS TO SPECIFIC SECTIONS OF THIS PAGE
2. LINKS BACK TO THE TOP OF THE PAGE
3. LINKS TO OTHER HTML FILES
4. ANCHORS FOR LINKS FROM PROFILE FOR
SPECIFIC SECTIONS OF THIS PAGE
5. ORDERED AND UNORDERED LIST
6. VARIED TYPES

!>
<html>
```

```
<head>
<title>resume of Joseph Andrew Gale</title>
</head>
<body>
<body bgcolor=gold>
<A NAME="TOP"></A>
<h1>Joseph Andrew Gale</h1>
<A HREF="pic.gif">CLICK HERE FOR PICTURE</A>
<h3>ADDRESSES</h3>
<PRE>
SCHOOL:
RCSU CAMPUS BOX #693
ELIZABETH CITY STATE UNIV.
ELIZABETH CITY, NC.27909
</PRE>
<OL>
<LI><H3><A HREF="#EMAIL">E-MAIL ADDRESSES</A></H3>
<LI><H3><A HREF="#PROFILE">PROFILE</A></H3>
<LI><H3><A HREF="#EDUCATIONAL">EDUCATIONAL EXPERIENCE</A></H3>
<LI><H3><A HREF="#ADVANCED">ADVANCED PROGRAMS</A></H3>
<LI><H3><A HREF="#ORGANIZATIONS">ORGANIZATIONS</A></H3>
<LI><H3><A HREF="#HOBBIES">HOBBIES</A></H3>
<LI><H3><A HREF="#INTEREST">INTEREST</A></H3>
<LI><H3><A HREF="#GOALS">GOALS & EXPECTATIONS</A></H3>
</OL>
<H3><A NAME="EMAIL">E-MAIL ADDRESSES</H3>
<OL>
<LI>j_gale@hotmail.com
<LI>j_gale@rocketmail.com
<LI>jgale@umfourt.cs.ecsu.edu
</OL>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<H3><A NAME="PROFILE">PROFILE</A></H3>
<OL TYPE="DISC">
<LI>PARENTS: DEBORAH Y. GALE & MIKE E. GALE
<LI>BIRTHDATE: JANUARY 8,1979
<LI>HEIGHT:6'1"
<LI>WEIGHT:170LBS.
<LI>ETHNIC: AFRICAN-AMERICAN
<LI>MARITAL STATUS:VERY SINGLE
</OL>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<A NAME="EDUCATIONAL"></A><H3>EDUCATIONAL EXPERIENCE</H3>
HIGH SCHOOL DIPLOMA FROM DOUGLAS MACARTHUR HIGH SCHOOL<br>
CURRENTLY A FRESHMAN AT ELIZABETH CITY STATE UNIVERSITY<br>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<A NAME="ADVANCED"></A><H3>ADVANCED PROGRAMS</H3>
<OL TYPE="I">
<LI>HONORS PROGRAM
<LI>NASA-ONR RESEARCHERS PROGRAM
</OL>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<A NAME="ORGANIZATIONS"></A><H3>ORGANIZATIONS</H3>
<OL>
<LI>TOP TEENS OF AMERICA
<UL>
<LI>ADMINISTRATIVE ASSISTANT
<LI>CHAIRMAN OF BEAUTIFICATION
</UL>
<LI>NACAP
<UL>
<LI>CO-CHAIRMAN OF MEMBERSHIP
</UL>
<LI>MARANATHA BAPTIST CHURCH
<UL>
<LI>YOUTH MINISTRIES
</UL>

```

HOME:  
4526 GUADALAJARA DR.  
SAN ANTONIO, TX.78233



```

</OL>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<A NAME="HOBBIES"></A><H3>HOBBIES</H3>
<UL TYPE = SQUARE>
<LI>BASKETBALL
<LI>ALL MUSIC
<LI>COMPUTERS
</UL>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<A NAME="INTEREST"></A><H3>INTERESTS</H3>
<UL>
<LI>BASKETBALL
<LI>ALL KINDS OF MUSIC
<LI>COMPUTERS
<LI>MOTORCYCLES
<LI>MOVIES
</UL>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A>
<A NAME="GOALS"></A><H3>GOALS & EXPECTATIONS</H3>
BACHELORS DEGREE IN COMPUTER SCIENCE<br>
MASTERS DEGREE IN COMPUTER ENGINEERING<br>
CAREER IN COMPUTER ENGINEERING FIELD AT NASA<br>
<A HREF="#TOP">CLICK HERE TO GO TO TOP</A><BR>
<A HREF = "PROFILE.HTML">CLICK HERE FOR PROFILE</A><BR>
<A HREF = "TABLE.HTML">CLICK HERE FOR TABLE</A>
</BODY>
</HTML>

```

```

<!--Joseph Gale
IN THIS PAGE WE INCORPORATED:
1. TABLES
2. CAPTIONS
3. BACKGROUND COLOR!>

```

```

<html>
<body bgcolor=white>
<CENTER>
<TABLE>
<caption><H1><blink><b>#CAMPUS NUMBERS#</b></blink></H1></CAPTION>
<TR>
<TH ALIGN = "LEFT">NAME</TH>
<TH ALIGN = "RIGHT">TELEPHONE</TH>
</TR>
<TR>
<TD>JOEY</TD>
<TD>8531</TD>
</TR>
<TR>
<TD>ANGELA</TD>
<TD>8432</TD>
</TR>
<TR>
<TD>ALBERT & GREG</TD>
<TD>8527</TD>
</TR>
<TR>
<TD>CANDACE & DONISE</TD>
<TD>8402</TD>
</TR>
<TR>
<TD>DEANNA</TD>
<TD>8427</TD>
</TR>

```

# *Visualization*



*Team Mentor: Dr. K. Edoh*

*Team Members: Kuchumbi Hayden  
Alicia Jones*

dimensional plots, and 3-dimensional plots from the gridded or irregular stored data at the Data Representation stage. The final stage is making animations from your graphical plots.

## A STUDY OF THREE NASA VISUALIZATION SOFTWARE

### INTRODUCTION

Visualization is a method of creating graphical representation of scientific data. It gives scientists a chance to see things that may not be recognized under normal circumstances. Visualization helps to enhance the scientific method of discovering the unknown.

It consists of the following methods:

Data File Formats;

Data Representation Formats;

Graphical Representation;

and Making Animations.

Data file format includes storing your data in one of the following formats:

- (1) ASCII Data
- (2) Binary Data
- (3) F77 Unformatted Binary Data
- (4) HDF format
- (5) CDF - The Common Data Format
- (5) NetCDF - Network Common Data Form
- (6) RGB - Format of Silicon Graphics
- etc.

The binary formats are machine dependent while the HDF format is a machine-independent binary data which has all of the advantages of binary data, and solves the problem of portability. The rest of the formats have their advantages and disadvantages. The data representation includes storing n-dimensional data in m-dimensions for graphical representation. The graphical representation involves creating 1-dimensional plots, 2-

There are many software packages that can be used to perform visualization. Among these are the three we studied; IISS, LINKWINDS, and WORLD WATCHER all of which were developed by NASA. We compared the first two software by looking at how they implemented the four methods of visualization discussed above. Like looking at the number of data formats accepted by each. This is to enable us to study how the two were developed. We also looked at the number of operating systems supported by each software and their networking capabilities. We studied WORLD WATCHER to learn how to create a similar tools for use by high school students. Throughout the study of these three software packages we used datasets provided by NASA to study each package and to figure out how they work.

### THE LINKED WINDOWS INTERACTIVE DATA SYSTEM

The Linked Windows Interactive Data System or LINKWINDS is a test bed that comes from NASA/JPL program of research. It transforms the research into graphs that makes it easy for faster and interactive accessing, displaying and analyzing large groups of datasets. LINKWINDS provide a variety of functions and services. Two examples include a 2-dimensional and 3-dimensional graph that displays data, interactive color manipulation, and animated displays of data. LINKWINDS operations allow the interconnection, through data linking, multiple windows containing visual displays and controls. This paradigm allows the system to function like a spreadsheet. This method allows the user to organize large

amounts of data for analysis. It also makes the system easy-to-learn and easy-to-retain user interface. The linking of data provides the ability to rapidly explore large masses of complex data to detect trends in the data. LINKWINDS is steadily being developed to get in close collaborative data with investigators in a variety of disciplines including oceanography, geography, geology, and cellular biology. LINKWINDS has the ability to ingest and display real time data.

## DATASETS

A dataset is a collection of values of one variable. A database is a collection of one or more datasets. LINKWINDS is able to accept data which is rectangularly gridded. A rank two dataset is an image. A rank three dataset is called a series of images. A rank four dataset is called a series of volumes. Higher ranked dataset are not named at this point. Only scalar data is accepted. The formats that are being accepted by LINKWINDS are:

- (1) Raw binary data - which includes single byte, signed and unsigned and unsigned shorts and longs.
- (2) The Hierarchical Data Format (HDF) of the National Center for Supercomputing Applications.
- (3) The Common Data Format (CDF) originated at Goddard Space Flight Center.
- (4) Network Common Data Format (NetCDF), derived form CDF at the National Center for Atmospheric Research.
- (5) The Silicon Graphics, Inc. native RGB image format.
- (6) Data with Planetary Data System (PDS) headers, originated at the Jet Propulsion Laboratory.
- (7) The Flexible Image Transport System (FITS) widely used by the astrophysics community.
- (8) ASCII text. As with raw binary data, a db file must also be supplied to give metadata.
- (9) Two data formats of UARS, the Upper Atmosphere Research Satellite. Both the format maintained by the Goddard DAAC and the UNIX variant of the format of the Central Data Handling Facility (CDHF) are readable.

- (10) The data format SAGE, the Stratospheric Aerosol and Gas Experiment.

## DATA LINKING

The key to using LINKWINDS is data linking. Data Linking gives the user the ability to link the many applications of LINKWINDS for concerted actions in examining data. It is effective in detecting trends in data and also correlations in data.

Data linking is affected through two icons. The link icon is a button displaying two interlocked rings, while the unlink icon displays two rings that are separated. To perform a link, the left mouse button is pressed while the cursor is placed on three appropriate button, and a "rubber band" type structure is dragged out and dropped into the application to be linked. To break the link, the same thing is done using the unlink button. The rubber bands indicating the current links may be displayed at any time during a session by the left-mouse button.

## INTERACTIVE IMAGE SPREADSHEET (IIS)

The IIS Environment, along with traditional numerical spreadsheets, enables the interactive organization, manipulation and processing of large amounts of data in a comprehensible manner. However, the IIS Environment uses three-dimensions to accommodate the sorting of data in a timely manner. Every page or layer of the spreadsheet is capable of manipulating an independent group of data. Along with manipulating data it has the capability of added flexibility of relationships between cells and layers. Each cell of the spreadsheet contains one or more

development and maintenance time. It is also considered essential to aid in program development.

multidimensional data set(s) to be visualized. A visualization data sets includes raw and processed satellite imagery, graphical data, surface and terrain models and three-dimensional volumes.

The image spreadsheet data structure contains all of the data required to define, display and manipulate a set of image data. Members of the data structure at the same level in the hierarchy have the same parent data field. The sheet is a two-dimensional array of Cells with each cell initially being the same screen dimension. Cells are arranged in a matrix form and are accessed via matrix addressing since the two-dimensional cells will normally remain stable during a user's session. Resizing an individual cell is not allowed, because it would upset the regular organization of the spreadsheet. However, rows and columns will be resizable since the matrix appearance will still be preserved under those operations. The image spreadsheet size, which is the number of rows and columns of cells, can also be dynamically changed by the user. More information from the Sheet level includes screen size of sheet, cells and colormap, titles or rows and columns, capabilities of zooming into an image by fractional amounts, hardware features and limitations.

Each cell contains one or more frames arranged as a linked list and denoted by the term Frame Stack. Two groups of pointers are maintained, one to keep track of all the frames in the Cell and another to keep track of the animation sequence when a subset of frames are looped together.

An effective user interface in this day and time should facilitate ease of use, consistency, portability, extensibility and maintainability. The use of a pre-existing library of user interface building blocks, is used to reduce

The Interactive Image Spreadsheet (IISS) Environment is a useful tool for developing quick image algorithms and browsing through large databases. The temporal browse feature for quickly examining large volumes of data. The current high performance hardware environment has made possible the effective manipulation and analysis of moderately large satellite data sets. It is anticipated that future improvements in CPU performance, disk I/O performance, network performance and access to multigigabyte random access memory will be necessary to fully take advantage of the capabilities of the IISS.

## WORLD WATCHER

WORLD WATCHER is a software package produced by NASA scientists to enable high school students the ability to emulate the work of actual scientists. WORLD WATCHER is based on the ClimateWatcher software package. It is aimed to provide an accessible and supportive environment for students to explore, interpret and analyze scientific data. WORLD WATCHER is similar to IISS and LINKWINDS because it uses a spreadsheet image. The only difference in WORLD WATCHER, LINKWINDS, and IISS is that WORLD WATCHER is only available for the Macintosh computer systems and PowerMacs.

## CONCLUSION

LINKWINDS, IISS and WORLD WATCHER are three visualization packages by NASA that use a spreadsheet format. All three packages use datasets. LINKWINDS and IISS can be ran on the Irix 4, SGI workstations, and the Sun computers. WORLD WATCHER is the only package that is limited to only computer, Macintosh. LINKWINDS uses the MUSE network operation system. IISS, however, uses Raw data.

## References

LinkWinds

<http://linkwinds.jpl.nasa.gov/>

Public Use of Remote Sensing Data

<http://rsd.gsfc.nasa.gov/rsd/>

- IISS

<http://rsd.gsfc.nasa.gov/rsd/IISS.html>

- Mr. K. Palaniappan

<http://rsd.gsfc.nasa.gov/users/palani/>

WorldWatcher

<http://www.covis.nwu.edu/sciviz/sciviz.html>

Dennis Chesters' Home Page

<http://climate.gsfc.nasa.gov/~chesters/chesters.html#goesproject>

EOSDIS

[http://eospsso.gsfc.nasa.gov/eos\\_homepage/eosdis.html](http://eospsso.gsfc.nasa.gov/eos_homepage/eosdis.html)

Bnet

<http://bernoulli.gsfc.nasa.gov/EBnet>

SI/NISN

<http://nic.nasa.gov>

REN

<http://www.nren.nasa.gov>

BNS

<http://www.vbns.net>

Internet2

<http://www.internet2.edu>

Image Spreadsheet

[http://rsd.gsfc.nasa.gov/rsd/images/iiss\\_cp.html](http://rsd.gsfc.nasa.gov/rsd/images/iiss_cp.html)

International Directory Network (IDN)

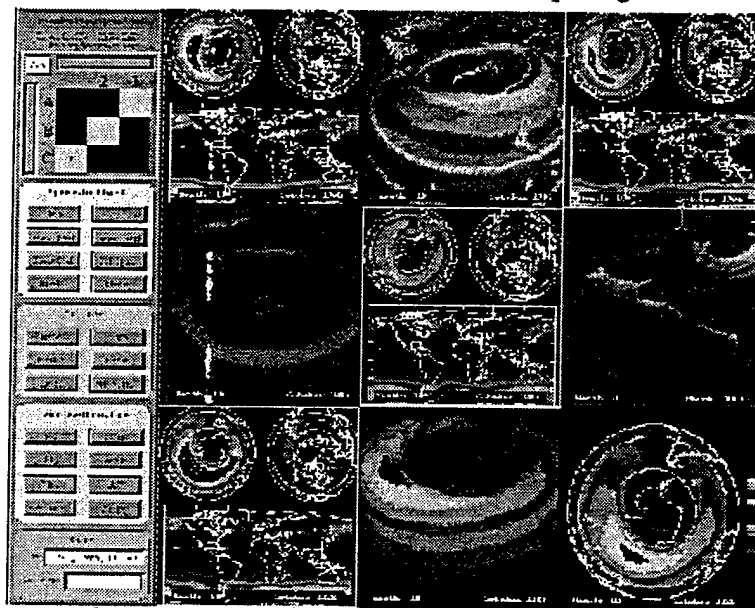
<http://gcmd.gsfc.nasa.gov/ceosidn>

## Earth Observing System (EOS) Distributed SpreadSheet

### NASA Sponsored

**Categories** Distributed Computing, Digital Libraries, Collaborative Computing

**Vision** Provide users with tools to perform advanced visualizations of very large datasets by the use of Interactive Image Spreadsheets. The Distributed Image SpreadSheet will provide access to terabyte Mission To Planet Earth (MTPE) archives, remote Earth Observing System (EOS) scientists, distributed tools and specialized hardware, and metacomputing resources.



**Why NREN?** The Distributed Image SpreadSheet (DISS) was developed by NASA Goddard Space Flight Center (GSFC) to provide users with tools to perform advanced visualizations of very large datasets. This type of tool is essential for scientists to allow them to analyze the large amount of data produced by next-generation satellite systems, such as EOS, that are expected to produce from 1 to 2 Terabytes of data per day.

**Description** The Distributed Image SpreadSheet (DISS) was developed by Dr. K. Palaniappan, now at the Univ. of Missouri Columbia (UMC), and Dr. Fritz Hasler of NASA Goddard Space Flight Center (GSFC). The DISS is based on a

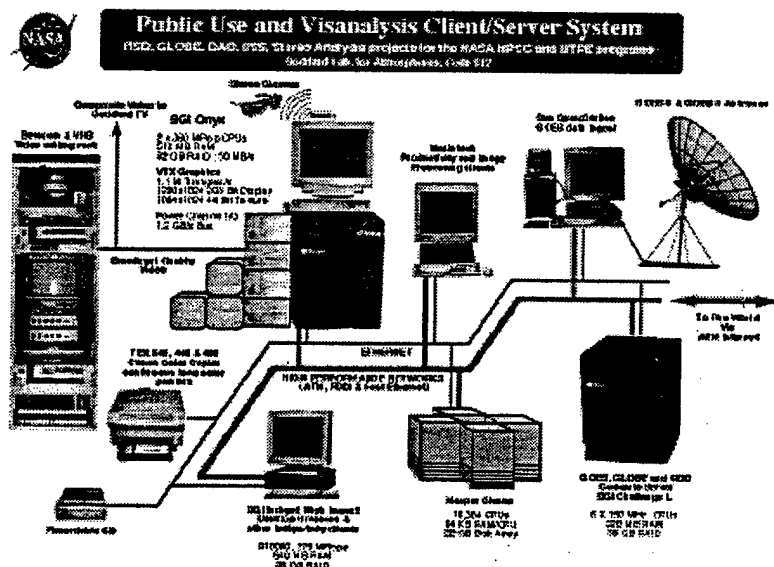
high performance compute engine and a user visualization terminal. The current version of the DISS is available to researchers for evaluation, and is under continuous development in the Laboratory for Atmospheres at NASA/GSFC and the Computer Engineering and Computer Science department at UMC. Network connectivity, testing, and performance evaluation was provided by the NASA Research and Education Network (NREN) and the Earth Science Data Information System (ESDIS) networking group at GSFC.

DISS is a highly interactive visualization and analysis (visanalysis) tool which combines a traditional spreadsheet paradigm with image processing, scientific visualization, and data archiving functionality. The DISS is a type of scalable visanalysis tool that is essential for scientists, enabling the analysis of the large amounts of hyper-image data produced by next generation satellite systems. NASA's contribution to Global Change Research is EOS, which is expected to have a total data production of 1 to 2 TB/day.

DISS provides users with tools to perform advanced visualizations of very large datasets such as the organization and comparison of almost 12 years of Total Ozone Mapping Spectrometer (TOMS) data. Other sample data sets used to show the development and evaluation of the functionality of the DISS include NOAA Advanced Very High Resolution Radiometer, NOAA Geostationary Operational Environmental Satellite, Landsat Thematic Mapper, Special Sensor Microwave/Imager, Airborne Visible and Infrared Imaging Spectrometer, NEXRAD, P3 aircraft radar, and EOS assimilated numerical model data produced by Goddard's Data Assimilation Office.

### *Technical Summary:*

#### Configuration of the Image Spreadsheet System (non-distributed)





### ***Demonstration:***

In June 1997 at the National Center for Atmospheric Research (NCAR) in Boulder, Colorado, DISS was demonstrated for the first time at the Global Observation Information Network (GOIN) workshop, a US-Japan bilateral demonstration of information networks. For this demonstration, the user client workstation (a remote visualization terminal operated at NCAR) accessed data remotely at GSFC (about 1500 miles away) from a high performance storage system via a high performance WAN. The WAN connectivity included a NASA NREN connection at OC-3 rates to NCAR via a vBNS connection at San Diego Supercomputing Center at OC-12 rates, and Fiber Distributed Data Interface (FDDI) network interfaces for the server and client.

In September 1997 at NASA Ames Research Center in Moffett Field, California, DISS was demonstrated using Asynchronous Transfer Mode (ATM) end-to-end for the first time. An OC-3 connection using NREN's network via the Sprint ATM Cloud was established between GSFC and Ames for the NREN Workshop. Application performance of the DISS dramatically improved by a factor of ten using ATM end-to-end.

### ***Schedule:***

The initial demonstration occurred June 23, 1997, at the NCAR in Boulder, Colorado, as part of the GOIN US-Japan initiative. A second demonstration at the NREN Workshop 2 (September 1997) successfully used ATM end-to-end between GSFC and ARC.

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### **Benefit**

Distributed visanalysis and high-speed remote access to geophysical data archives via high performance networks is critical to the success of NASA's Mission to Planet Earth program. The Goddard ESDIS project collaborates with the Ames NREN project for the purpose of prototyping EOS Data Information System (EOSDIS) applications. The DISS is an EOS application that is expected to be widely used by the EOS community. The DISS tool is important to other agencies involved in data analysis and modeling for aeronautics, astrophysics, geophysical studies, environmental monitoring, and mapping. The DISS tool with NREN connectivity facilitates public and commercial use of meteorological datasets; for example, broadcast weather services such as NBC WRC-TV already use NOAA's satellite imagery processed by NASA using this system on a daily basis.

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### **Requirements**

DISS has several modes of distributed operations. In one scenario, the user visualization terminal is remote from the compute engine, and connected via a high performance WAN (e.g., NREN) which is used for communicating graphics rendering information. In another configuration, the compute engine accesses remote data over a high performance network using distributed file systems or direct client-server protocols. These distributed modes of operation may also be combined and used with Web technologies (e.g., Java). Initial testing and operation of the DISS will need effective network performance of at least 100 Mbps. Eventually, the server will have the potential to operate at near gigabit and higher speeds when pulling in data from multiple remote sources or serving multiple collaborating distributed users.

Networked visual analysis (visanalysis) requirements include:

- Bandwidth: 50 MB/s access to 100 GB "Redundant Array of Independent Discs"
- Latency: Low latency (< 0.5/second for interactive visualization and responsiveness)
- Data: Distributed MTPE datasets and archives
- Collaboration: Distributed Image SpreadSheet visualization with client and server sharing workload
- Video: Novel virtual environments and distribution of multimedia results

#### Partners and Potential Partners

Partners will include other government agencies involved in atmospheric analysis, including NOAA and NCAR.

#### URLs

- <http://rsd.gsfc.nasa.gov/rsd/>
- <http://rsd.gsfc.nasa.gov/rsd/IISS.html>
- <http://globe.gsfc.nasa.gov/globe>
- <http://rsd.gsfc.nasa.gov/users/palani/>

To download DISS for SGI:

<ftp://agnes.gsfc.nasa.gov/pub/iiss>

To contact Dr. Palaniappan:

<http://rsd.gsfc.nasa.gov/users/palani>

To locate more information on NREN and its applications:

<http://www.nren.nasa.gov>



#### Partners in NGI

- Dept. of Defense  
<http://www.dod.mil/dod-www.html>  
<http://www.crl.mil/HPCMP/DREN/>
- Defense Advanced Research Projects Agency  
<http://www.darpa.mil/ResearchAreas.html>
- DoD's Energy Science Network  
<http://www.es.net>
- Internet 2  
<http://www.internet2.edu/>
- National Aeronautics and Space Administration  
<http://www.nasa.gov>  
<http://www.nren.nasa.gov>
- National Institute of Health  
<http://www.nih.gov/>
- National Institute of Standards and Technology  
<http://www.nist.gov/>
- National Oceanic and Atmospheric Administration  
<http://www.noaa.gov/>
- National Science Foundation  
<http://www.nsf.gov>  
<http://www.cis.nsf.gov/ncrl/bp-connections.html>  
<http://www.xbs.net>



**Next Generation Internet**

- Promote experimentation with the next generation of networking technologies.
- Develop a next generation network testbed to connect universities and Federal research institutions at rates that demonstrate new networking technologies and that support future research.
- Demonstrate new applications that meet important national goals and missions and that rely on the advances made in goals above.



# TIME LOOPS FROM A HIGHER ORDER GRAVITATIONAL THEORY

A. L. CHOUDHURY

Department of Physical Sciences  
Elizabeth City State University  
Elizabeth City, N.C. 27909  
U.S.A.

E-mail: choudhal@alpha.ecsu.edu

## Abstract

In the extended Coule and Maeda version of the Gidding-Strominger model of the gravitational field interacting with an axion, we showed in an earlier communication that using zeroeth and first order perturbation terms and a special pasting technique we can create time loops. Here we show that the incorporation of a second order perturbation term yields the same results.

## 1. INTRODUCTION

Using the Coule-Maeda' extension of the Gidding-Strominger model<sup>1</sup> we have formulated a perturbative procedure<sup>2</sup> to calculate the scale factor and the scalar field function. We have used the procedure in an earlier paper<sup>3</sup> to compute the first order scale factor. We showed there that if we could superimpose the solutions and paste them in a definite procedure we could construct time loops.

We extended this concept here to incorporate second order terms to calculate scale factors. In the process we ran into improper integrals with divergent contributions. We introduced a subtraction procedure to suppress such divergent terms by adjusting the constant of integration. We would like to interpret it as a new kind of subtraction closely analog to the renormalization procedure of modern field theory. However, in field theory the renormalization can be given a physical meaning, we are yet to give any similar interpretation in our case.

Carrying out the subtraction prescription, we have obtained a finite contribution for the infrared limit of the solution. We have then shown that we can construct time loops as we have done in the previous paper<sup>4</sup>.

We have collected the important results of the earlier paper CH2 in section 2. In section 3 we have carried out the computation of the second order scale factor  $a_2$  and the scalar field  $\phi_2$  in its lowest order computation. In section 4, we give the formal solution of  $a_2$ . We have introduced there an infrared cut-off  $\delta$  and computed the limiting function  $a(0)$ . Following the pasting technique of CH2, we have shown how to construct time loops using the same superimposition technique of that paper.

In section 5, we carried out a different superimposition and discussed the conditions for loop construction. In section 6, we discuss the results. In Appendix A we have shown the justification of a wormhole solution when we go over to the ultraviolet limit. In Appendix B, we show how we can suppress divergent terms and extract finite information from the solution.

## 2. WORMHOLE SOLUTION UPTO THE FIRST ORDER

We follow the prescription of CH1<sup>3</sup> to obtain the higher order terms of wormhole solutions. In order to make the paper self-contained, we quote in brief the essential formulae needed to discuss our solution. The equations we need to solve are (Eqns.(2.1) and (2.2) in CH2)

$$\frac{d^2 \phi}{dt^2} - \beta D e^{2\phi} = \frac{\partial U}{\partial \phi} a^4 \quad (2.1)$$

and

$$\left( \frac{1}{a} \frac{da}{dt} \right)^2 - a^4 + a_2^4 = -\frac{\kappa^2}{3} U(\phi) a^2. \quad (2.2)$$

We choose

$$U(\phi) = -\lambda_1 \phi^2. \quad (2.3)$$

As mentioned in CH1,  $\tau$  is related to the real time  $t$  by the equation

With Corrections  
to appear in *Modern Physics*

$$dt = a^{-1} dt.$$

(2.4)

At the zeroeth order in Eqn.(2.2) we neglect the right hand side term and obtain the wellknown solution

$$z_0^2 = z_c^2 \sec(2z_c^{-1} \tau) \quad (2.5)$$

which can be written as

$$z_0 = \pm z_c \sqrt{\sec(2z_c^{-1} \tau)}. \quad (2.5a)$$

In the first order perturbation Eqn.(2.2) takes the form

$$\frac{dz_1}{dt} - g(\tau) z_1 = Q(\tau) \quad (2.6)$$

where

$$Q(\tau) = \frac{\kappa^2 \lambda_1}{6a_c} \phi_0^2 z_0^2 \cos(2z_c^{-1} \tau) \quad (2.7)$$

and

$$g(\tau) = z_c^{-1} [2 \cos(2z_c^{-1} \tau) + 3 \tan(2z_c^{-1} \tau)]. \quad (2.8)$$

In Eqn.(2.7),  $\phi_0$  is the zeroeth order solution of Eqn.(2.1) when the right hand side of the equation is ignored. One of the solutions is given by

$$\phi_0(\tau) = -\frac{1}{\kappa} \frac{1}{z_c} \ln(K_0^{-1} \cos(2z_c^{-1} \tau)). \quad (2.9)$$

Introducing a new variable

$$x = 2z_c^{-1} \tau \quad (2.10)$$

and writing

$$\tilde{a}_1(x) = z_1(\tau) \quad (2.11)$$

we get

$$Q_{11}(\tau) = \mp \frac{z_c^2}{2} \left[ \frac{1.5 \cos^{3/2}(2z_c^{-1} \tau) + \cos^{1/2}(2z_c^{-1} \tau)}{\sin(2z_c^{-1} \tau)} \right] \quad (3.3a)$$

$$\tilde{a}(x) = \frac{\lambda_1 z_c}{48} \frac{\sin x}{\cos^3 x} F(x, \delta) \quad (2.12)$$

where

$$\begin{aligned} |F(x, \delta)| &= \cot x (\ln(K_0^{-1} \cos x))^2 - \cot \delta (\ln(K_0^{-1} \cos \delta))^2 \\ &\quad - 2 \{ x \ln(K_0^{-1} \cos x) - \delta \ln(K_0^{-1} \cos \delta) \} - \{ F(x) - F(\delta) \}. \end{aligned} \quad (2.12a)$$

and

$$\begin{aligned} F(x) &= \int_0^x \tan x' dx' \\ &= \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^{2k} (2^{2k-1} - 1)}{(2k+1)!} B_{2k} x^{2k+1}. \end{aligned} \quad (2.13)$$

In the above equation  $B_{2k}$  stands for Bernoulli numbers. The essential criterion of  $F(x)$  is the fact that  $F(\pi/2)$  can be assumed to be finite as justified in paper CH2.

### 3. SECOND ORDER WORMHOLE SOLUTION

The second order equation for the scale factor is given by

$$\frac{dz_2(\tau)}{dt} - g(\tau) z_2(\tau) = Q_2(\tau) \quad (3.1)$$

where  $g(\tau)$  is given by the Eqn.(2.8) and

$$Q_2(\tau) = Q_{11}(\tau) + Q_{21}(\tau) + Q_{22}(\tau). \quad (3.2)$$

In Eqn.(3.2) the  $Q_2(\tau)$  are given by the following relations:

$$Q_{\pi}(\tau) = \frac{1}{2} \left[ \frac{14 \cos^n(2a_c \tau)}{\cos^{1/2}(2a_c \tau)} + \frac{2 \cos^{2n}(2a_c \tau)}{\sin(2a_c \tau)} + \frac{2 \cos^{2n}(2a_c \tau)}{\sin(2a_c \tau)} \right] + \frac{2}{a_c} \left[ 3 \sin(2a_c \tau) \cos^n(2a_c \tau) + \frac{2 \cos^{2n}(2a_c \tau)}{\sin(2a_c \tau)} \right] Q(\tau) + \frac{1}{2a_c} \frac{\cos^{2n}(2a_c \tau)}{\sin(2a_c \tau)} Q'(\tau) \quad (3.3b)$$

and

$$Q_{\pi}(\tau) = -\frac{\kappa^2}{6} \frac{\Phi_0 a_0}{\lambda_1} \frac{1}{a_c} \left\{ \frac{8 \Phi_0 a_0}{\tan(2a_c \tau)} + \frac{2 \Phi_0 a_0}{\tan(2a_c \tau)} \right\} \quad (3.3c)$$

We use the new variable  $x$  defined in Eqn.(2.10) and introduce new variables  $\bar{Q}(x)$  and  $\bar{Q}_r(x)$  in future computations defined as follows:

$$\bar{Q}(x) = Q(\tau) \quad (3.4)$$

$$Q_r(\tau) = \bar{Q}_r(x) \frac{1}{2a_c} = \left[ \bar{Q}_{\pi}(x) + \bar{Q}_{\pi}(x) + \bar{Q}_{\pi}(x) \right] \frac{1}{2a_c} \quad (3.5)$$

and

The new  $\bar{Q}$  and  $\bar{Q}_r$  are then given by the following relations:

$$\bar{Q}(x) = \pm \frac{\lambda_1 a_0^2}{24} \left[ \ln(K_0^{-1} \cos x) \right]^2 \sec^{2n} x, \quad (3.6)$$

$$\bar{Q}_{\pi}(x) = \mp \frac{\bar{a}_1(x)}{4a_c} \left[ \frac{15 \cos^{2n} x + \cos^{2n} x}{\sin x} \right], \quad (3.7)$$

$$\bar{Q}_{\pi}(x) = \mp \frac{\bar{a}_1(x)}{4a_c} \left[ \frac{12 \sin x}{\cos^{1/2} x} + \frac{14 \cos^{2n} x}{\sin x} + \frac{2 \cos^{2n} x}{\sin^3 x} \right] + \frac{\bar{a}_1(x)}{a_c} \left[ 3 \sin x \cos^{2n} x + \frac{2 \cos^{2n} x}{\sin x} \right] \bar{Q}(x) + \frac{1}{2a_c} \frac{\cos^{2n} x}{\sin x} \bar{Q}'(x), \quad (3.8)$$

and

$$\bar{Q}_{\pi}(x) = -\frac{\kappa^2 \lambda_1}{12a_c} \frac{\Phi_0 a_0}{\lambda_1} \frac{1}{a_c} \left[ \frac{8 \Phi_0 a_0}{\tan x} + \frac{2 \Phi_0 a_0}{\tan x} \right] \frac{1}{\tan x} \bar{Q}(x) + \frac{2 \Phi_0 a_0}{\tan x} \bar{Q}'(x). \quad (3.9)$$

The zeroeth order of the first order solution  $\Phi_1^{(0)}(\tau)$  has been derived by the author in paper CH1 as given in Eqn.(A.29) there. The quantity  $\bar{\Phi}_1^{(0)}(x)$  in Eqn.(3.10) is related to that solution by the following relation:

$$\Phi_1(\tau) \equiv \Phi_1^{(0)}(\tau) = \frac{\bar{\Phi}_1^{(0)}(x)}{(2a_c)^{1/2}} \quad (3.10)$$

We can therefore write

$$\bar{\Phi}_1^{(0)}(x) = -\cosh \bar{f}(x) \int_0^x dx' \frac{x' \sinh \bar{f}(x')}{\bar{f}(x')} \bar{H}(x') + \frac{x \sinh \bar{f}(x)}{\bar{f}(x)} \int_0^x dx' \cosh \bar{f}(x') \bar{H}(x') \quad (3.11)$$

In deriving the Eqn.(3.11) we have chosen the boundary condition at  $\tau = 0$

$$\Phi_1(\tau) = 0 \quad (3.12)$$

The constants in Eqn.(A.29) in the paper CH1 yields

$$\Phi_{10} = 0, \quad X_{10} = 0 \quad (3.13)$$

In Eqn.(3.11) the function  $\bar{f}(x)$  and  $\bar{H}(x)$  have the following forms

$$\bar{f}(x) = f(\tau) = \frac{1}{a_c} \sqrt{\Delta x \tan x} \quad (3.14a)$$

with

$$\Delta = \left( \frac{K_0 D}{2} \right)^{1/2} \beta = 2a_c^2 \omega \quad (3.14b)$$

and

$$\bar{H}(x) = H(\tau) = -\frac{a_c^2}{12 \kappa^{1/2}} \frac{\ln(K_0^{-1} \cos x)}{\cos^2 x} \quad (3.15)$$

We also define  $g_0(x)$  as follows:

$$g_0(x) = g(\tau)/(2a_0^2) = \cot x + \frac{3}{2} \tan x. \quad (3.16)$$

With all these newly introduced functions we can convert the Eqn.(3.1) into the following form:

$$\frac{d\tilde{a}_1(x)}{dx} - g_0(x)\tilde{a}_1(x) = \tilde{Q}_1(x). \quad (3.17)$$

Hence the solution for the scale function can be written as

$$\begin{aligned} \tilde{a}_1(x) &= \text{Exp} \left( \int g_0(x) dx \right) \left[ \text{Exp} \left( - \int g_0(x) dx \right) \tilde{Q}_1(x) + C' \right] \\ &= s(x) \left[ \int s^{-1}(x) \tilde{Q}_1(x) dx + C' \right]. \end{aligned} \quad (3.18)$$

In the above equation we have performed the integration of  $g_0(x)$  and obtained

$$\int g_0(x) dx = \frac{\sin x}{\cos^2 x} = s(x). \quad (3.19)$$

To avoid the blow up of the integrand at  $x=0$ , we now introduce a cut-off  $\delta$ . Hence the final form of  $\tilde{a}_1(x)$  can be written as

$$\tilde{a}_1(x) = s(x) \left[ \int_0^x s^{-1}(x') \tilde{Q}_1(x') dx' + C_0 \right] \quad (3.20)$$

where the constant  $C_0$  can depend on  $\delta$ . We now claim that the derived solution of  $\tilde{a}_1(x)$  can be extended to the value  $x \rightarrow 0$ . In the process of going to this limit any divergent term appearing during the process of integration can be subtracted out by the constant  $C_0$ . The leftover terms will be finite. We show the rationale of the claim in Appendix B. In order to justify that our total solution is a wormhole solution upto the second order term we still have to show that for  $x \rightarrow \pi/2$  the total  $\tilde{a}(x) \rightarrow \infty$ . We will justify this conclusion in Appendix A.

#### 4. TOTAL SOLUTION UPTO THE SECOND ORDER.

Collecting the zeroeth, the first order, and the second order terms of  $\tilde{a}(x)$  from Eqns. (2.5a), (2.12), and (3.20), we get

$$\tilde{a}(x) = \tilde{a}_0(x) + \tilde{a}_1(x) + \tilde{a}_2(x) \quad (4.1)$$

where we have set  $\tilde{a}_0(x) = a_0(\tau)$ . We now go to the limit  $x \rightarrow 0$  (which is equivalent to  $\tau \rightarrow 0$ ) in Eqn.(4.1). Referring to CH2 we find

$$\tilde{a}_0(0) = 2a_0, \quad (4.2)$$

and

$$\tilde{a}_1(0) = \frac{2a_0 \lambda_1 (\ln K_0)^2}{48}. \quad (4.3)$$

We now want to extract the limit of  $\tilde{a}_i(x)$  as  $x \rightarrow 0$ . We introduce several abbreviations in  $\tilde{a}_i(x)$ . Defining

$$I_{12}(x) = s(x) \int_0^x s^{-1}(x') [\tilde{Q}_1(x') + \tilde{Q}_{12}(x')] dx', \quad (4.4)$$

and

$$I_3(x) = s(x) \int_0^x s^{-1}(x') \tilde{Q}_{12}(x') dx', \quad (4.5)$$

we can write

$$\tilde{a}(x) = I_{12}(x) + I_3(x) + C_0 s(x). \quad (4.6)$$

We can further split  $I$ 's as follows:

$$I_{12}(x) = s(x) [A_1(x) + A_2(x) + A_3(x)] \quad (4.7)$$

where

$$\begin{aligned} A_1(x) &= \frac{\lambda_1^2 a_0^2}{2^2 3^2} T_1(x) \\ &= \frac{\lambda_1^2 a_0^2}{2^2 3^2} \int_0^x dx' \left\{ \frac{12 \sin^2 x'}{\cos^2 x'} + 14 + \frac{2 \cos^2 x'}{\sin^2 x'} \right\} |F'(x', \delta)|, \end{aligned} \quad (4.8)$$

$$A_1(x) = \frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} T_1(x)$$

$$= -\frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} \int_0^x \frac{dx'}{\cos x'} \left\{ \frac{3 \sin x'}{\cos x'} + \frac{1}{\sin x' \cos x'} \right\} \left\{ \ln(K_0^{-1} \cos x') \right\}^2 |F(x', \delta)|, \quad (4.9)$$

and

$$A_3(x) = \frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} T_3(x)$$

$$= -\frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} \int_0^x \frac{dx'}{\sin x' \cos^2 x'} \left\{ \ln(K_0^{-1} \cos x') \right\}^2. \quad (4.10)$$

The term  $I_4(x)$  in Eqn.(4.6) can be split as follows:

$$I_4(x) = q(x) [B_1(x) + B_2(x)] \quad (4.11)$$

where

$$B_1(x) = \frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} T_4(x)$$

$$= -\frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} \int_0^x \frac{dx'}{\sin x' \cos^2 x'} \left\{ \ln(K_0^{-1} \cos x') \right\}^2 |F(x', \delta)|, \quad (4.12)$$

and

$$B_2(x) = \frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} T_5(x)$$

$$= +\frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} \int_0^x \frac{dx'}{\sin x' \cos^2 x'} \left\{ \frac{\ln(K_0^{-1} \cos x')}{\sin x' \cos x'} \right.$$

$$\left. + \frac{\int_0^x \frac{\ln(K_0^{-1} \cos x'')}{\cos x''} \left[ -x'' \sinh \tilde{f}(x') \cosh \tilde{f}(x') + \frac{x' \sinh \tilde{f}(x') \cosh \tilde{f}(x')}{\tilde{f}(x')} \right]}{\tilde{f}(x')} \right\}. \quad (4.13)$$

All  $T_i(x)$ -s defined in Eqns. (4.8)-(4.13) are expressions for which we have evaluated the limits to  $x \rightarrow 0$  in Appendix B. We finally obtain

$$\tilde{a}_1(0) = \lim_{x \rightarrow 0} \tilde{a}_1(2\delta) = \frac{\lambda_1^2 \lambda_2^2}{2 \cdot 3^2} (\ln K_0)^4 \quad (4.14)$$

Since  $\tilde{a}_0(x)$ ,  $\tilde{a}_1(x)$ , and  $\tilde{a}_2(x)$  are solutions of certain differential equations, we can form the most general solution by superimposing them according to our choice. We can thus construct as we have done in CH1( we will refer to this as Case A)

$$\tilde{a}^A(x) = \tilde{a}^A a(x) + \tilde{a}^A u(x) + \tilde{a}^A x(x) \quad (4.15)$$

with the choice

$$\tilde{a}^A a(0) = \lim_{x \rightarrow 0} \tilde{a}^A a(x) = -a_c, \quad (4.16)$$

$$\tilde{a}^A u(0) = \lim_{x \rightarrow 0} \tilde{a}^A u(x) = -\frac{a_c \lambda_1 (\ln K_0)^2}{42}, \quad (4.17)$$

and

$$\tilde{a}^A x(0) = \lim_{x \rightarrow 0} \tilde{a}^A x(x) = -\frac{a_c^2 \lambda_1^2 (\ln K_0)^4}{1152}. \quad (4.18)$$

Therefore, upto second order at  $x=0$   $\tilde{a}_1(x)$  becomes

$$\tilde{a}_1^A(0) = a_c + \frac{a_c \lambda_1 (\ln K_0)^2}{42} + \frac{a_c^2 \lambda_1^2 (\ln K_0)^4}{1152}. \quad (4.19)$$

We can similarly construct

$$\tilde{a}_2^A(x) = \tilde{a}^A a(x) + \tilde{a}^A u(x) + \tilde{a}^A x(x) \quad (4.20)$$

with the limiting condition at  $x=0$ ,

$$\tilde{a}_2^A a(0) = a_c, \quad \tilde{a}_2^A u(0) = -\frac{a_c \lambda_1 (\ln K_0)^2}{42}, \text{ and}$$

$$\tilde{a}_2^A x(0) = \frac{\lambda_1^2 a_c^2 (\ln K_0)^4}{1152}. \quad (4.21)$$

Therefore,

$$\tilde{a}_1^A(0) = a_c - \frac{a_c \lambda_1 (\ln K_0)^2}{42} + \frac{\lambda_1^2 a_c^2 (\ln K_0)^4}{1152}. \quad (4.22)$$

Thus we see

$$\bar{a}^1(0) < \bar{a}^1(0) \quad (4.23)$$

We can then follow the procedure of CH2 to paste the solutions and hence construct the time loops as shown in the diagram in Figure 1.

## 5. A NATURAL SUPERPOSITION

In the previous chapter we have superimposed the solutions of the differential equations according to the prescription of the paper CH2. Pursuing a different path we can define

$$\bar{a}^2(x) = \bar{a}^0(x) + \bar{a}^1(x) + \bar{a}^2(x) \quad (5.1)$$

We will refer to it subsequently as Case B. In the limit  $x \rightarrow 0$ , we get

$$\bar{a}^2(0) = -a_e + \frac{a_2 \lambda_1 (\ln K_0)^2}{42} + \frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} \quad (5.2)$$

We can designate

$$\bar{a}^2_f(0) = a_e - \frac{a_2 \lambda_1 (\ln K_0)^2}{48} + \frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} \quad (5.3)$$

and

$$\bar{a}^2_f(0) = -a_e + \frac{a_2 \lambda_1 (\ln K_0)^2}{48} + \frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} \quad (5.4)$$

$\bar{a}^2(x)$  is defined as the function which yields  $\bar{a}^2_f(0)$  as we approach  $x \rightarrow 0$ , whereas we designate  $\bar{a}^2(x)$  to be the function which leads to  $\bar{a}^2(0)$  in the same limit. Both  $\bar{a}^2_f(0)$  and  $\bar{a}^2(0)$  have to be assigned positive values in order to give a physical meaning to each of the quantities interpreting them as the radii of the space in that limit. In order to do that we must simultaneously satisfy two conditions

$$1 - \frac{\lambda_1 (\ln K_0)^2}{48} + \frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} > 0 \quad (5.5)$$

and

$$1 - \frac{\lambda_1 (\ln K_0)^2}{48} - \frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} < 0 \quad (5.6)$$

Both conditions can be simultaneously satisfied if

$$-\frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} < 1 - \frac{\lambda_1 (\ln K_0)^2}{48} < \frac{\lambda_1^2 a_2^2 (\ln K_0)^4}{1152} \quad (5.7)$$

The difference between  $\bar{a}^2(0)$  and  $\bar{a}^2_f(0)$  is given by

$$\bar{a}^2(0) - \bar{a}^2_f(0) = 2a_e \left[ 1 - \frac{\lambda_1 (\ln K_0)^2}{48} \right] \quad (5.8)$$

If we choose

$$\frac{\lambda_1 (\ln K_0)^2}{48} < 1 \quad (5.9)$$

we find

$$\bar{a}^2_f(0) > \bar{a}^2(0) \quad (5.10)$$

If we can now show that  $\bar{a}^2_f(x)$  and  $\bar{a}^2(x)$  goes to  $+\infty$  as  $x \rightarrow \pi/2$ , we can carry out the pasting procedure described in CH2 and create a time loop as shown in Figure 2.

Combining the restriction of  $\lambda_1$  in Eqn.(5.9) with that in Eqn.(A.13), we find that  $\lambda_1$  has to satisfy the inequality

$$\frac{96\pi}{24\pi^2 + 85} < \lambda_1 < \frac{48}{(\ln K_0)^2} \quad (5.11)$$

## 6. CONCLUDING REMARKS

When we deal with higher order perturbation terms to calculate the scale factor  $\bar{a}(x)$  we run into divergence problems. We have analyzed the



nature of the divergences in Appendices A and B. In our case we are dealing with two kinds of divergences. The first type is an infrared divergence when we go to the limit  $x \rightarrow 0$ . The other type is an ultraviolet divergence when we go to the limit  $x \rightarrow \pi/2$ . We do not concern ourselves with the second kind, when we look for wormhole solutions because the scale function has to blow up to positive infinity. The only criterion that we have to watch is the fact that the scale function  $\tilde{a}(x)$  has to have a positive signature for the limit  $x \rightarrow \pi/2$ . We have checked in Appendix A what condition  $\lambda_i$  must satisfy to have the right signature for the scale factor. The Eqn. (B.13) gives us the bound. We are however more concerned with the infrared divergences. Such divergences appear in functions  $T_i(x)$  for  $i = 1$  through 5 defined in the Eqns. (4.8) through (4.13). Most of the divergences show up due to the appearance of  $\sin x$  in the denominators. We have initially evaded such divergences formally by introducing a  $\delta$  cut-off. However, we need to go to a sensible limit  $x \rightarrow 0$  for a meaningful  $\tilde{a}(x)$  at the neck of the wormhole. In order to eliminate the divergence in that limit we have utilized the integration constant  $C_0$  which appeared in Eqn. (3.21). We defined this constant in such a way that it cancels the divergent terms and some constant terms from the infrared limits and makes the remaining terms symmetric with lower order terms. This type of subtraction is not unknown in field theories. The theory of renormalization is in fact such a subtraction procedure. However in field theories we can associate a physical meaning with the concept of renormalization by attaching a dressed particle property obtained from a bare particle existence. We are aware of the fact that this interpretation is meaningless here because we have to deal with quantities which are basically infinite. The procedure none-the-less has a solid foundation because the leftover terms yield measurable expressions which have been tested experimentally.

We are not claiming any such interpretation for two reasons. Firstly the theory does not describe a Lorentzian world and secondly we are not sure whether a wormhole solution has any physical existence. But our procedure of divergence elimination might show us a way to tackle similar problems in an equivalent real theory. After subtraction we get a wormhole solution which has a finite value when we go to the limit  $x \rightarrow 0$ . In section 4, we have shown that if we follow the author's earlier paper CH2, we can generate a time loop shown in Figure 1. To reach this conclusion we have assumed that different solutions of a particular order can be superimposed.

In section 5 we have kept the sign tags at proper places at every solution. In this procedure the first order solution does not create any loop. But the addition of the second order term enables us to construct a loop as shown in Figure 2.

We admit that presently the above prescription to suppress divergence contribution has no physical interpretation. But we hope that in future we can extract some meaningful insight if we can use the procedure in a theory of the real world. We are now looking for such insight in a real theory of wormholes.

#### APPENDIX A

In this appendix we intend to show that the solution upto the second order of  $\tilde{a}(x)$  yields positive infinity to retain the interpretation of a wormhole. In order to give  $\tilde{a}(x)$  finite value as we approach  $x \rightarrow \pi/2$ , we introduce an ultraviolet cut-off  $\delta'$ . All the integrations run from  $\delta$  to  $\pi/2 - \delta'$ . From the definition of  $IF(x, \delta)$  (Eqn. (2.12a)), we can easily obtain dominant terms in the neighborhood of  $\pi/2 - \delta'$ , remembering  $\cos(\pi/2 - \delta') = \delta'$  and  $\sin(\pi/2 - \delta') = 1$  for small  $\delta'$ ,

$$IF\left(\frac{\pi}{2} - \delta, \delta\right) = -\left(\frac{(\ln K_0)^2 + \pi \delta \ln \delta'}{\delta}\right) \quad (A.1)$$

From the definition of  $T_1(x)$

$$\begin{aligned} T_1\left(\frac{\pi}{2}\right) &= \lim_{x \rightarrow 0} T_1\left(\frac{\pi}{2} - \delta'\right) \\ &= \lim_{x \rightarrow 0} \int_{\delta}^{\pi/2 - \delta'} dx' \left\{ \frac{12 \sin^2 x'}{\cos^2 x'} + 14 + \frac{2 \cos^2 x'}{\sin^2 x'} \right\} F^2(x', \delta) \end{aligned} \quad (A.2)$$

the divergent contribution can be isolated if we look into the term

$$\begin{aligned} \text{Dominant Div. Terms} &= \int_{\delta}^{\pi/2 - \delta'} dx' \left\{ \frac{12 \sin^2 x'}{\cos^2 x'} + 14 + \frac{2 \cos^2 x'}{\sin^2 x'} \right\} F^2(x', \delta) \\ &= \left\{ \frac{12 \sin^2\left(\frac{\pi}{2} - \delta'\right)}{\cos^2\left(\frac{\pi}{2} - \delta'\right)} + 14 + \frac{2 \cos^2\left(\frac{\pi}{2} - \delta'\right)}{\sin^2\left(\frac{\pi}{2} - \delta'\right)} \right\} \left( \frac{(\ln K_0)^2 + \pi \delta \ln \delta'}{\delta} \right) \delta' \\ &= \frac{24\pi^2}{\delta'} \end{aligned} \quad (A.3)$$

Therefore the dominant term in  $s\left(\frac{\pi}{2}-\delta'\right)T_1\left(\frac{\pi}{2}\right)$  is  $D_{11}$  where

$$D_{11} = s\left(\frac{\pi}{2}-\delta'\right)T_1\left(\frac{\pi}{2}\right) = \frac{24\pi^2}{(\delta')^{3\pi}} \quad (\text{A.4})$$

Similarly we can show that the dominant term in  $s\left(\frac{\pi}{2}-\delta'\right)T_2\left(\frac{\pi}{2}\right)$  is  $D_{12}$  where

$$D_{12} = \frac{36}{(\delta')^{3\pi}} \quad (\text{A.5})$$

Associated with  $T_3$ , we have the dominant term  $D_{13}$  where

$$D_{13} = \frac{48}{(\delta')^{3\pi}} \quad (\text{A.6})$$

For  $T_4$ , we have

$$D_{14} = \frac{96\pi}{\lambda_1} \frac{1}{(\delta')^{3\pi}} \quad (\text{A.7})$$

We can now show that the dominant term of  $s\left(\frac{\pi}{2}-\delta'\right)T_5\left(\frac{\pi}{2}\right)$  can be expressed as

$$D_{15} = \frac{16\pi^2 d(\pi/2)}{3\lambda_1} \frac{1}{(\delta')^{3\pi}} \quad (\text{A.8})$$

The constant  $d(\pi/2)$  is given by the expression

$$d(\pi/2) = \frac{\pi \sinh[(\pi/2) \coth(\pi/2)]}{2 \Gamma(\pi/2)} \quad (\text{A.9})$$

On the other hand we have

$$D_{15} = \frac{16\pi^2 d(\pi/2)}{3\lambda_1} \frac{1}{(\delta')^{3\pi}} \quad (\text{A.10})$$

If we add them together they cancel one another. Thus

$$D_{15} = D_{13} + D_{15} = 0 \quad (\text{A.11})$$

We can easily generalize the statement that they cancel out upto the term proportional to  $(\delta')^{-3\pi}$ . If we add all terms together, we get

$$D_7 = \sum_{i=1}^7 D_{1i} = \left(24\pi^2 + 84 - \frac{96\pi}{\lambda_1}\right) \frac{1}{(\delta')^{3\pi}} \quad (\text{A.12})$$

To obtain a positive  $\tilde{a}(x)$  at  $\pi/2$  we have to introduce an extra condition which  $\lambda_1$  must satisfy. The new condition is

$$\lambda_1 > \frac{96\pi}{24\pi^2 + 85} \quad (\text{A.13})$$

## APPENDIX B

Here we demonstrate how we have carried out the limit  $x \rightarrow 0$ . We start to study the behavior of the function  $T_1(x)$  defined in the Eqn.(4.8). It is given by

$$T_1(x) = \int_0^{\frac{\pi}{2}} dx' \left[ \frac{12 \sin^2 x'}{\cos^2 x'} + 14 + \frac{2 \cos^2 x'}{\sin^2 x'} \right] |F^2(x', \delta)| \quad (\text{B.1})$$

As we have mentioned earlier we have introduced an infrared cut-off  $\delta$  to make the integration finite. The source of divergence originates from  $(1/\sin x)$  and  $\cot x$  of  $|F(x, \delta)|$  in the integrand. In order to go to the limit  $x \rightarrow 0$ , we evaluate  $T_1(x)$  for  $x=2\delta$  and then find the limit  $\delta \rightarrow 0$ , that is we are seeking the limit

$$\lim_{\delta \rightarrow 0} s(2\delta) T_1(2\delta) \quad (\text{B.2})$$

In Eqn.(B.1) we expand the integrand as power series of  $x'$  because  $x' < 2\delta$  is essentially a small quantity. The expansion upto second order in  $x'$  for  $|F^2(x, \delta)|$  can be shown to be

$$|F^2(x, \delta)| = L^2 \frac{(x-\delta)^2}{x^2 \delta^2} + \left( \frac{L^2 + 9L}{3} \right) (x-\delta)^2 \quad (\text{B.3})$$

where we have set

$$L = \ln K_0 \quad (B.4)$$

Upon integration  $T_1(2\delta)$  can be expressed as

$$T_1(2\delta) = \frac{37L^2}{12\delta^3} + O(\delta) \quad (B.5)$$

Therefore the expression  $s(2\delta)T_1(2\delta)$  comes out as

$$s(2\delta)T_1(2\delta) = \frac{37L^2}{6\delta^3} + O(\delta^2) \quad (B.6)$$

The above expression is quadratically divergent in the infrared limit. Similar computations can be carried out for other  $T$ 's. We find

$$s(2\delta)T_2(2\delta) = L^2(1 - 2\ln 2) + O(\delta) \quad (B.7)$$

$$s(2\delta)T_3(2\delta) = 2L^2 + O(\delta^2) \quad (B.8)$$

$$s(2\delta)T_4(2\delta) = \frac{2^3}{\lambda_1} \{L^2(1 - 2\ln 2)\} + O(\delta) \quad (B.9)$$

and

$$s(2\delta)T_5(2\delta) = O(\delta^2) \quad (B.10)$$

Now going back to the Eqn.(4.6) we find that for small  $\delta$ , in the case of  $x=2\delta$ ,  $\tilde{z}_1(x)$  becomes

$$\tilde{z}_1(2\delta) = \frac{\lambda_1^2 z_1^3}{2 \cdot 3} \left[ \frac{37L^2}{6\delta^3} + L^2(1 - 2\ln 2) + 2L^2 + \frac{2^3}{\lambda_1} \{L^2(1 - 2\ln 2)\} + O(\delta) + C_0 2\delta \right] \quad (B.11)$$

We still have a constant assumed to be dependent on  $\delta$ . We will use this constant  $C_0$  to subtract the divergent term away in the same way we subtract the divergences in quantum electrodynamics. We define

$$C_0 2\delta = \left[ \frac{37L^2}{6\delta^3} + L^2(1 - 2\ln 2) + 2L^2 + \frac{2^3}{\lambda_1} \{L^2(1 - 2\ln 2)\} \right] \quad (B.12)$$

We intentionally kept the term  $2L^2$  in the parenthesis of Eqn.(B.11) to retain the symmetric appearance of  $L^2$  in  $\tilde{z}_1(0)$ . We therefore obtain

$$\tilde{z}_2(0) = \frac{\lambda_1^2 z_2^3}{2 \cdot 3} L^4 \quad (B.13)$$

This is the expression we used in Eqn.(4.12).

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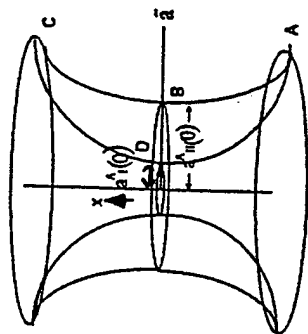


Figure 1.

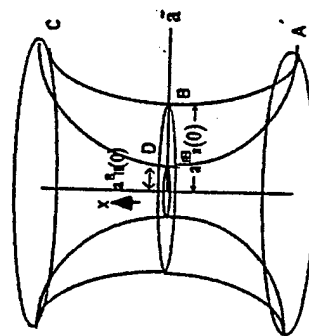
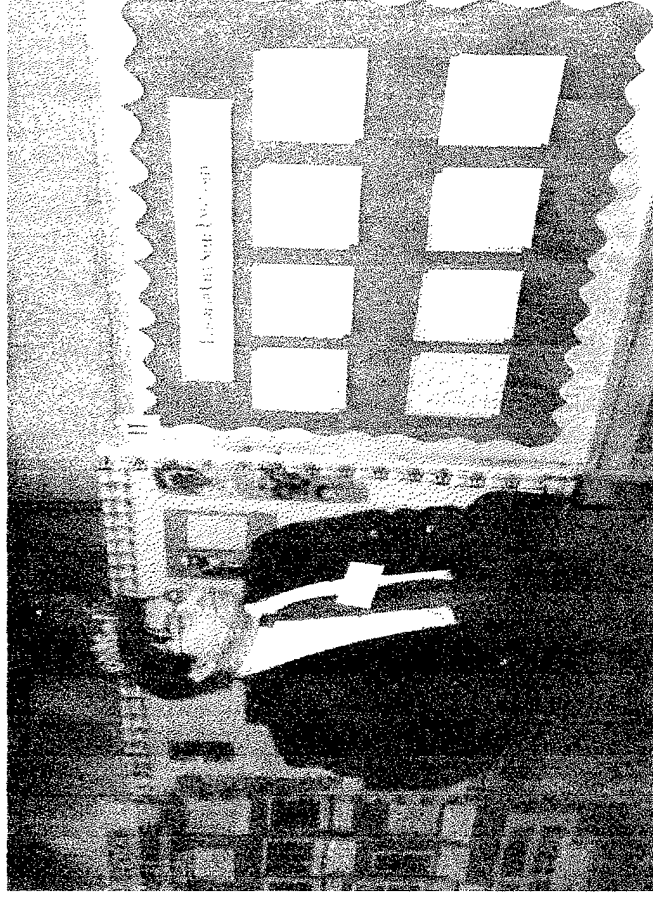


Figure 2.

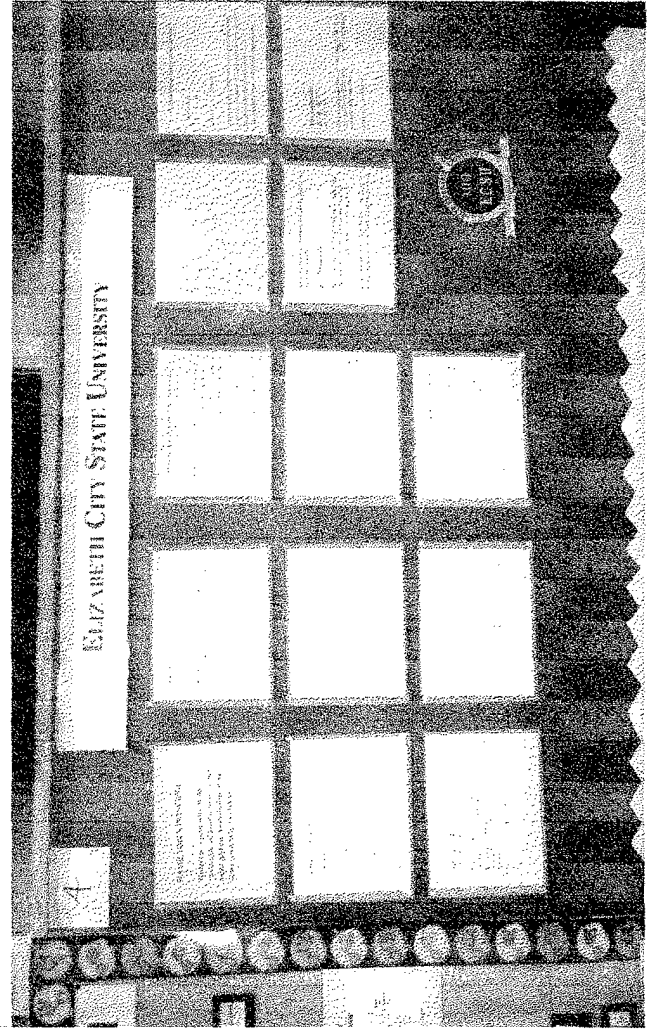
Figure Captions:

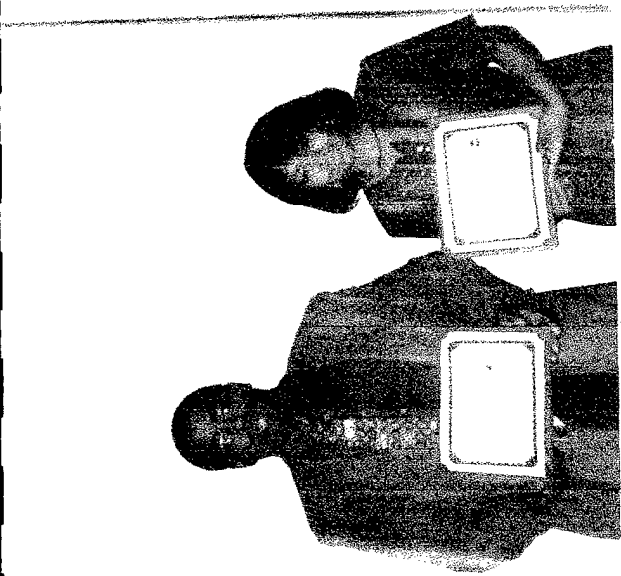
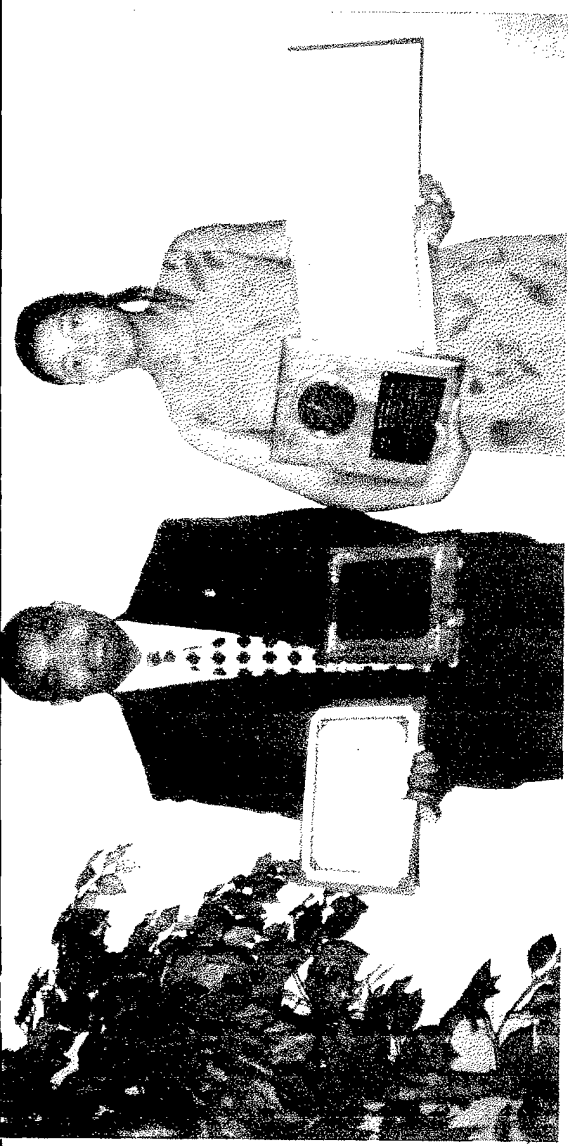
Figure 1. Time loop ABCDA in Case A.

Figure 2. Time loop ABCDA in Case B.

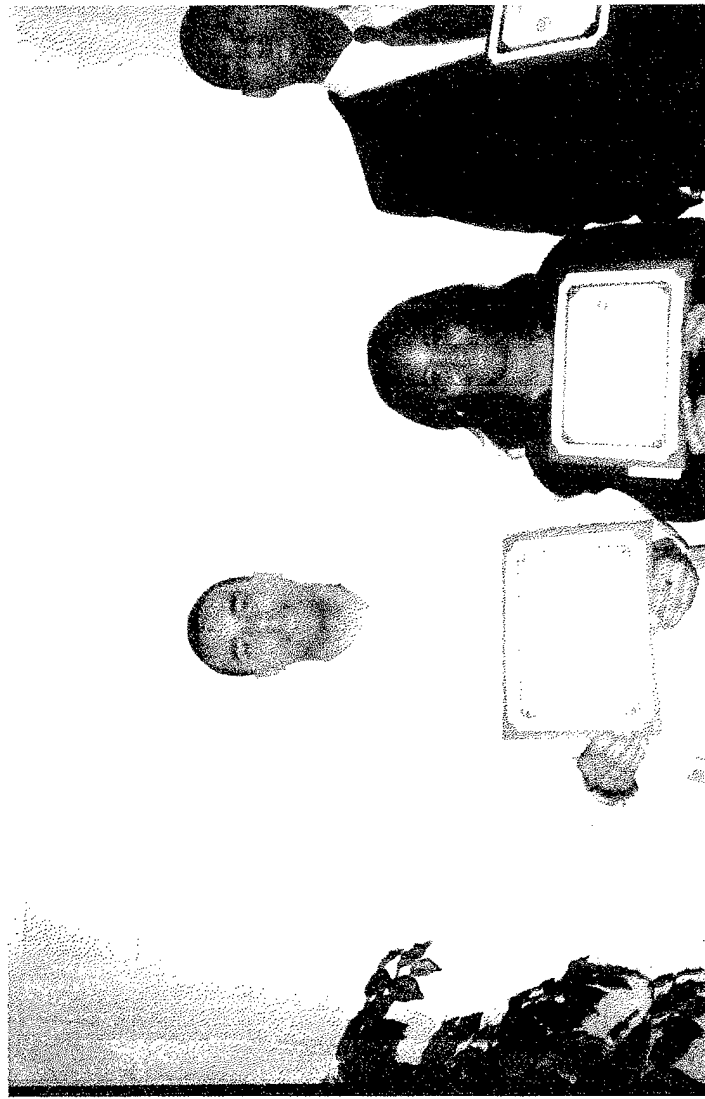
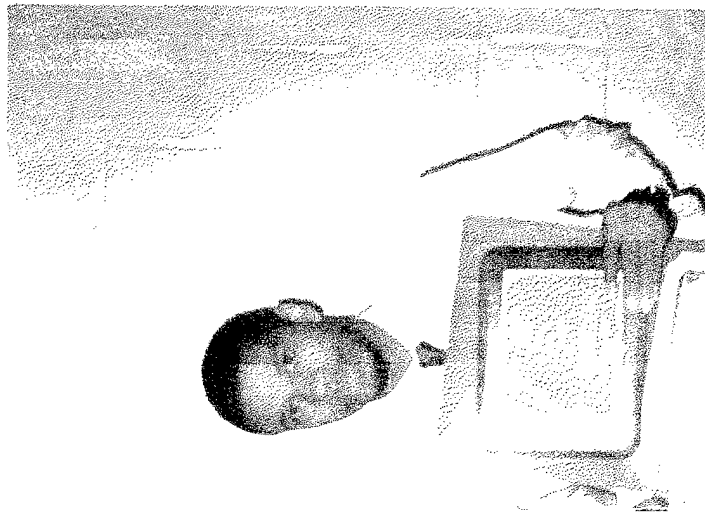


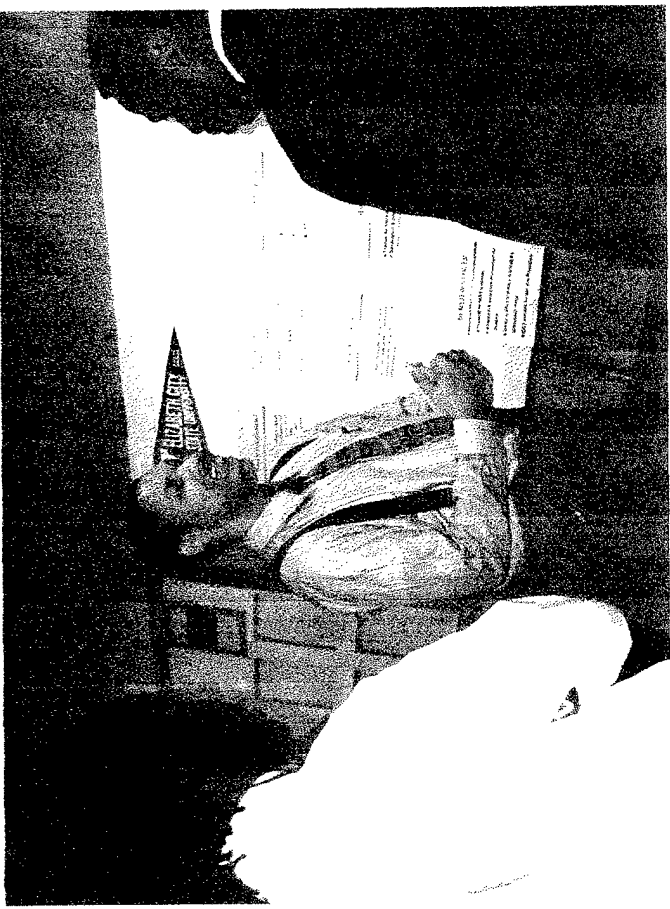
Highlights  
 1998 Department of Energy HBCU Research Symposium  
 Ocean City, Maryland April 26-29, 1998



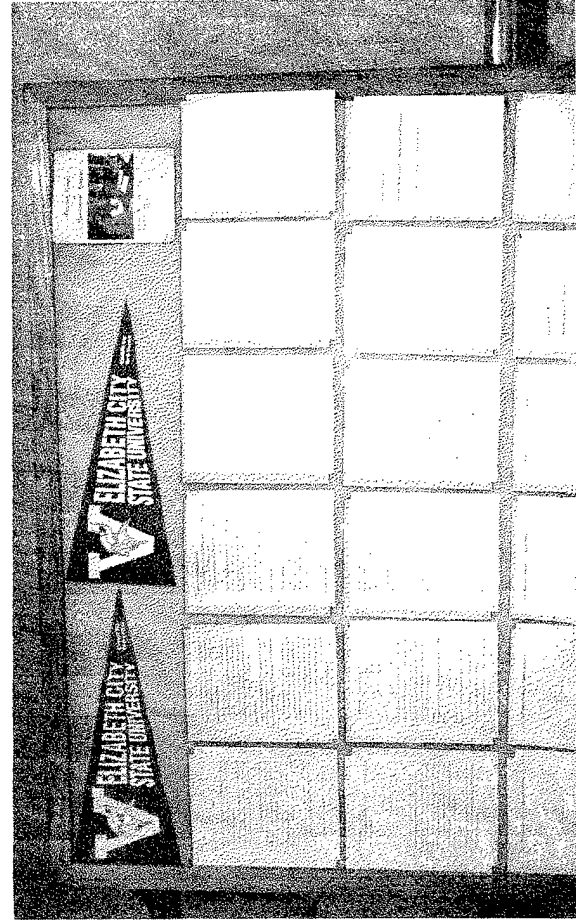
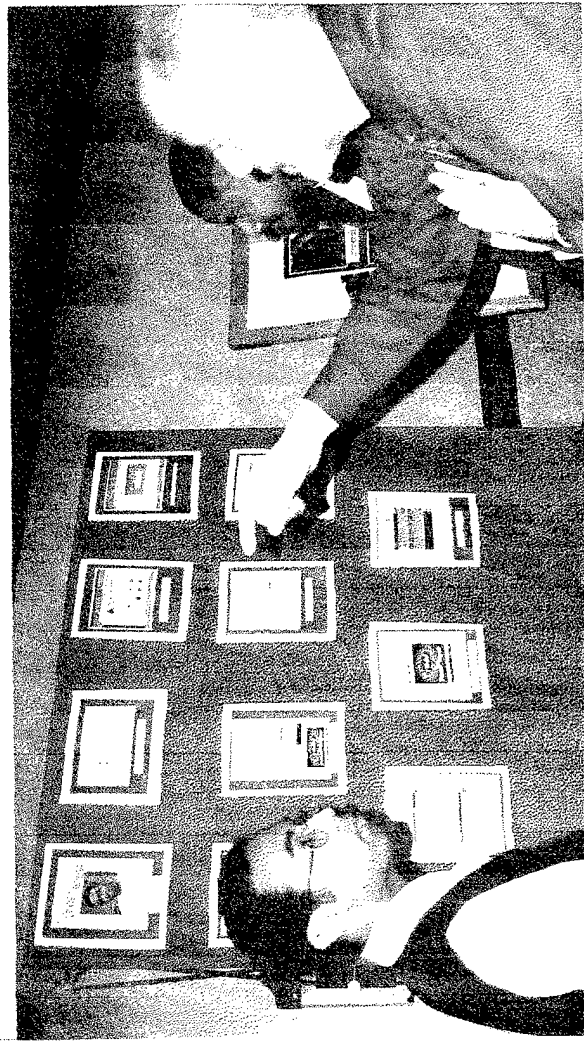


Highlights  
1998 Honors and Awards Day  
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 1998 NAFEO High Tech Undergraduate Student Expo  
 Washington, DC April 17, 1998

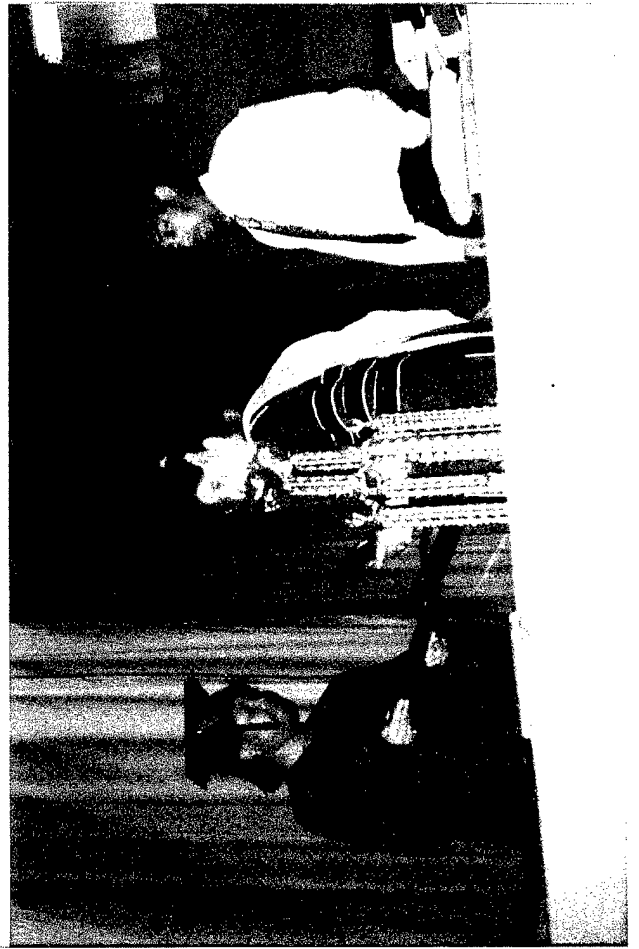




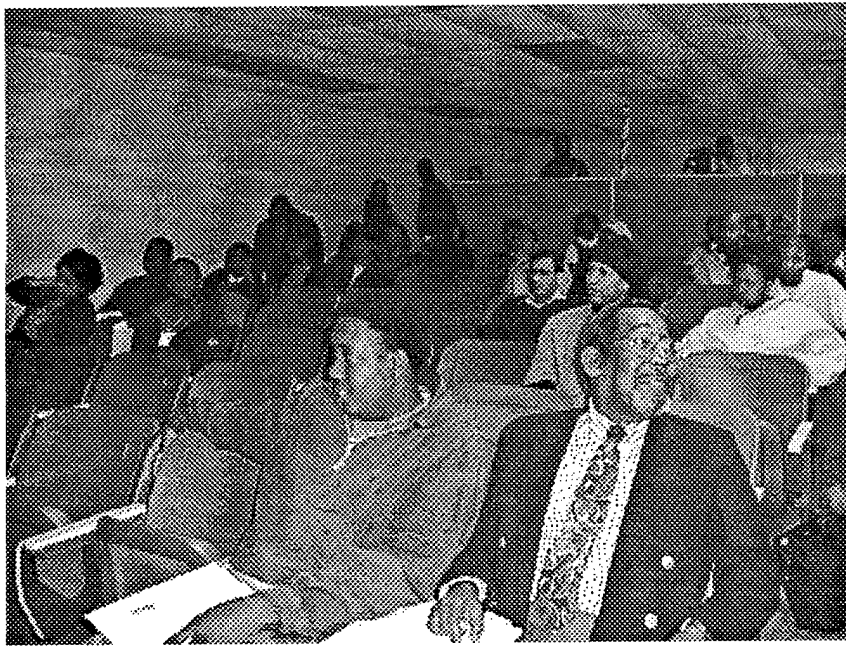


Above: Dr. Hayden is shown during the Press Conference to announce ECSU as the first HBCU to commercialize a NASA technology. Also shown below is Brian Jordan (Math Graduate Student-HU) as he discusses graduate school experience with Laverne Williams (Senior CS Major - 1998 GEM Fellow)

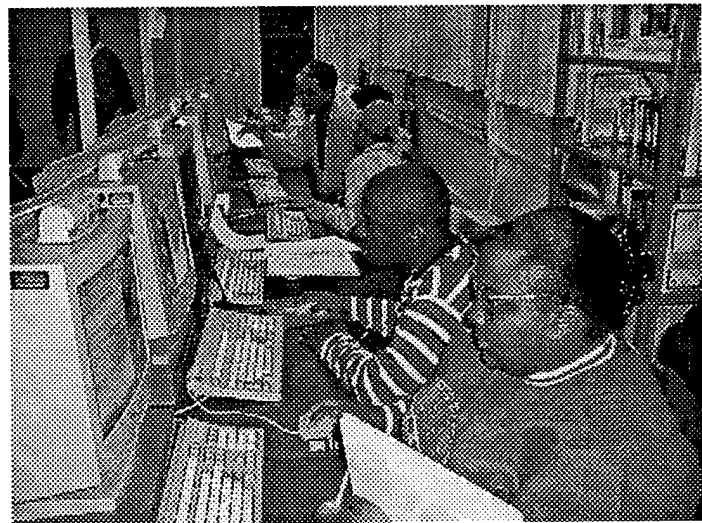
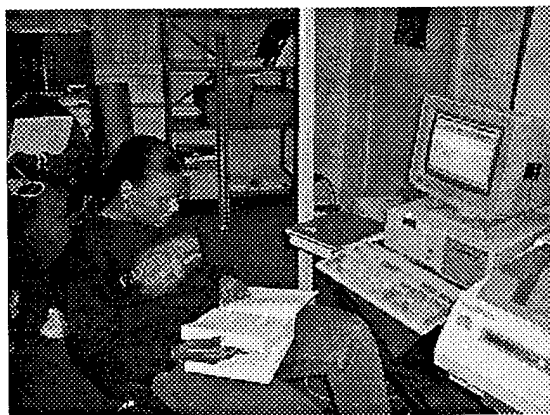
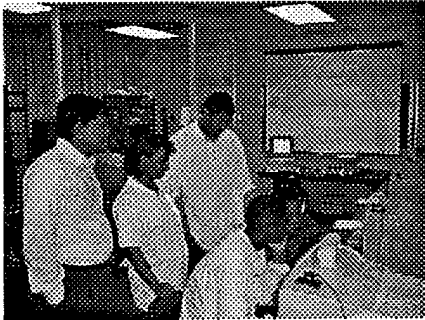
Below: During Honors and Awards Day, Corey Ellis (right) and Charles Gatling accept awards.





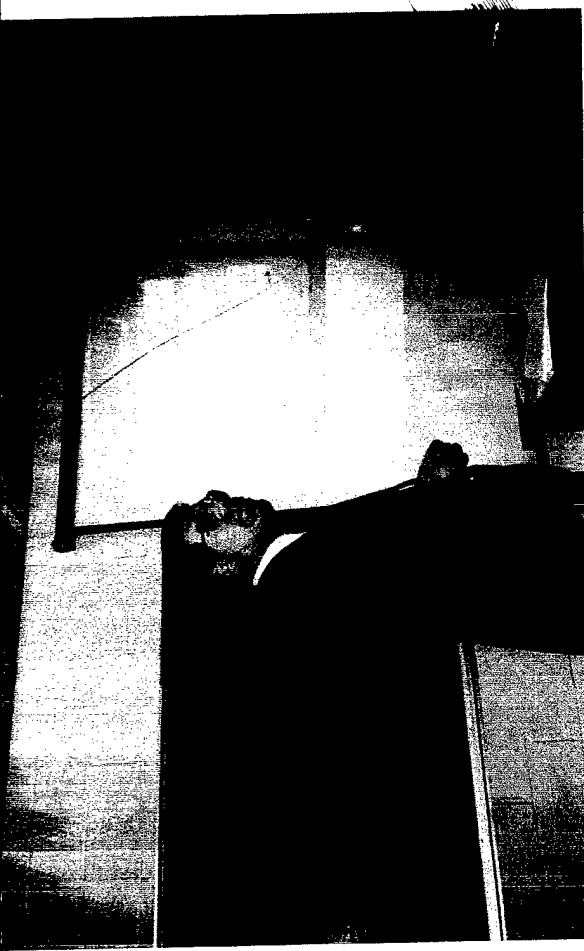


# 1997-98 Research Training Highlights



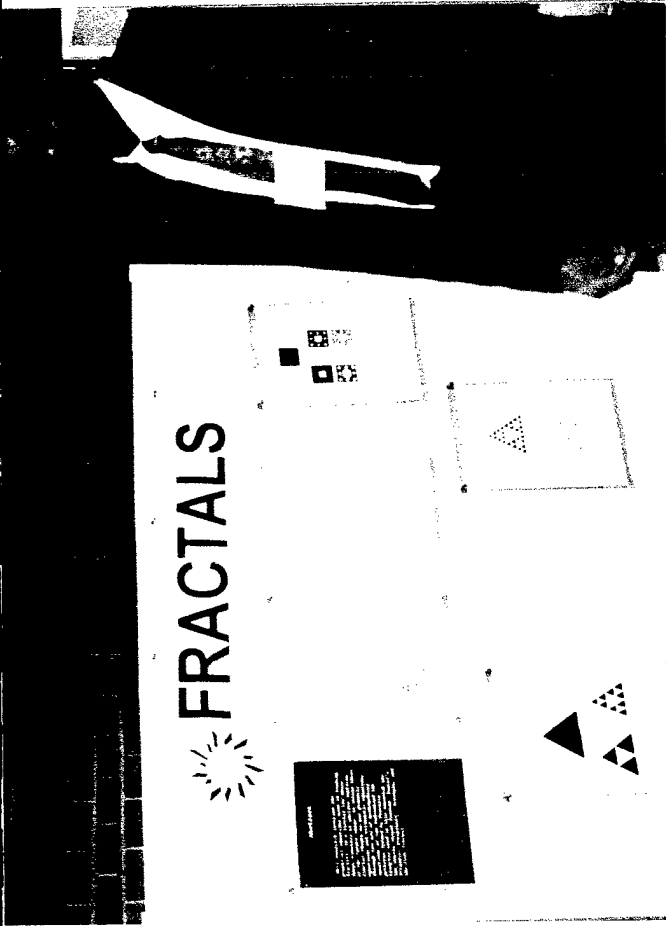


1997-98 Highlights



Visiting Lecture - Next Generation Internet (NGI)  
Andy Germaine, Engineer, Swales Aerospace Inc.

# Greensboro Hilton



1997-98 Highlights



# 1997-98 Research Training Highlights

